Chuong

-Bf-IT -o-A nc T-H LrC VR

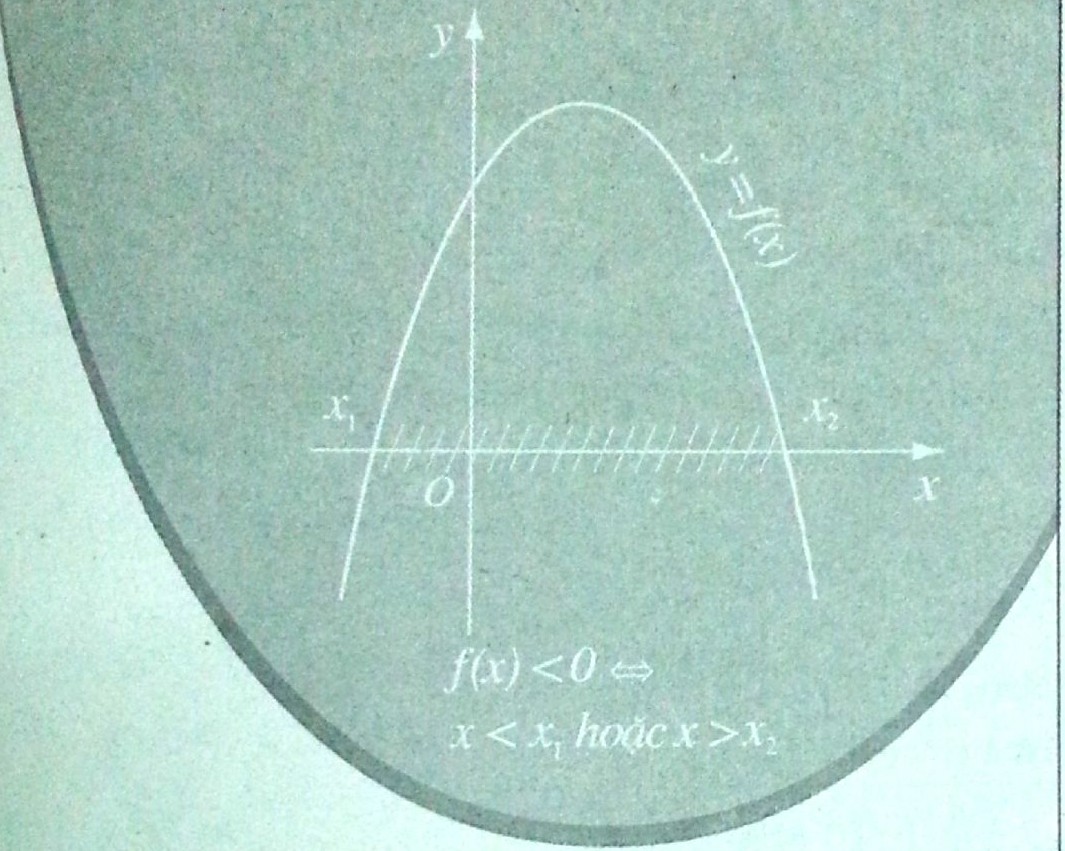
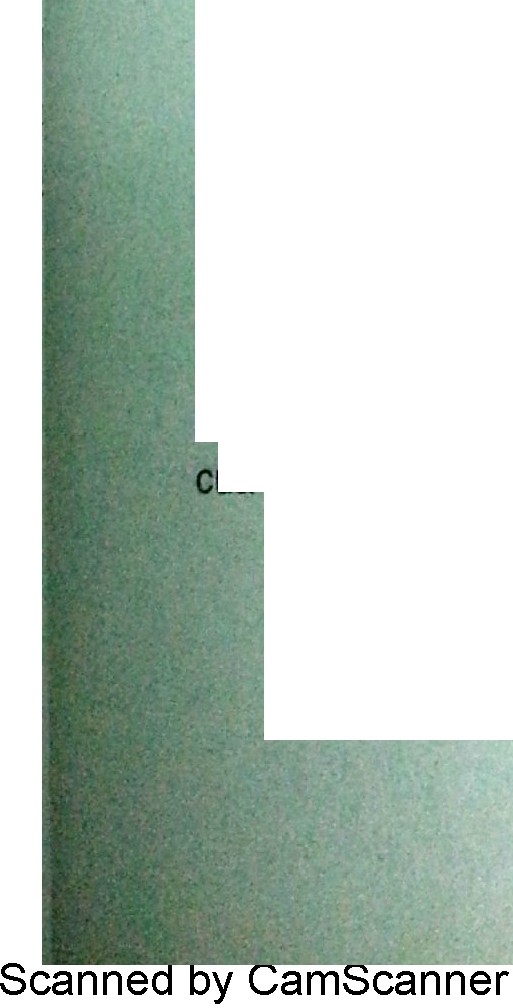
-8-AT -P H LfCTn G T-Rl n-H

*x*

*l y* =*j(x)*

- 0 + 0 -

... -- ---.....J



Bat d ng thuc va bat phuong trlnh la nhCrng khai nim ma chung ta da lam quen 6 lop duoi. Chuong nay se hoan thi$n

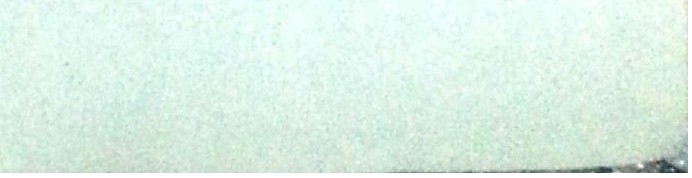
hon cac khai ni$m d6,dong thoicung cap cho chung ta nhCmg kiSnthuc moinhuvan de xet dau cua nhi thuc b$c nhat va dau rua tam thuc b$c hai.Chung c6 nhieu Crng dng quantrc;mg trong

vltc glai va bin lu$n cac phuong trinh va bat phuong trlnh.

Chung ta can nam vung cac kienthuc d6, d6ng thoi rem luy$n

1d nang ap dng chung de giaj cac bai toan trong khu6n kho

.1.11"!1 chuong trinh.



103

;

?

BAT DANG THVC



... ,, "" ' ?

VA CHUNG MINH BAT DANG THUc .

1. On tp va b6 sung tinh chat ciia bat dAng thuc

Gia Slr *a* va *b* Ia hai *s6* thtfc. Cac mnh de *"a >.b", "a < b", "a h", "a S b'*

duqc g<?i lanhiing *bat ddng th1lc.*

Cung nhu cac mnh de logic khac, m(>t bat ddng thuc c6 th6 *dilng* hoc *sai.*

ChUn.g minh 1n¢t bat dng thuc la cht!ng minh bat dfing thuc d6 dung.

Duoi day la m(>t *s6* tfnh chat da biet cua bat dang thuc.

*a* > *b* va *b* > *c* => *a* > *c*

*a > b* <=> *ci + c > b + c*

Neu *c* > 0 thl *a > b* <=> *ac > he.*

Neu *c* < 0 thl *a > b* <=> *ac < be.*

Tu d6 ta c6 cac h qua sau :

*a* > *b* va *c* > *d* => *a* + *c* > *b* + *d* ;

*a + c > b ¢=> a > b - c ;*

*a* > *b* > 0 va *c* > *d* > 0 => *ac* > *bd* ;

*a* > *b* > 0 va */1* E N\* => *a11* > *h11* ;

. *a* > *b* > 0 ¢::> >. *Jb* ;

*a* > *b* ¢::>. *v-;* > *lfb.*

Vi d1;1 1.Khong dung bang sohoc may tfnh, hay so sanh hai s6 *J2* + *J3* va J.

*Gidi.* Gia str .J2 + *J3* < J. Do hai ve cua bllt dltng thuc d6 deu <ltrcmg 060

*Ji* + *J3* < 3 *¢:::>* (*.J2* + *J3* )2 < 9 <=> 5 + 2 *J6* < 9

<=> 2*J6* < 4 <=> *J6* < 2 <:> 6 < 4, vO If.

Vy *J2* + *J3* > 3.

104 

. .

•'

.

. .

Scanned by CamScanner



Neu *A, B* Ia nhiing bieu thuc chua bien th'i *"A > B"* Ia m(> m\_nh chua ien... ChU'ng minh bat dlmg thuc *A > B* (vai dieu kin nao d6 cua cac b1en), nghia la chU'ng minh mnh de chti'a bien *A* > *B* dung vai tat ca cac gia tr!cua cac bien (thoa man dieu kin d6). · ·



Tir nay, fa quy uac : Khi n6i ta c6 bat dng thc *A* > *B* (trong d6 *A* va *B* Ia (.

nhihlg biu thuc chua bien) ma khong neu dieu kin d6i vai cac bien th'i ta t ;

I

hiu rAng bat dng thuc d6 xay ra vai m9i gia tr!cua bien thuqc JR. :

Vl°d 2. Chl1ng minh rang x2 > 2(x -l).



*Gidi. x* > 2(.t"-1) <::::> x2 > *2x -2* <::::> .x2 *-2x* +2 >·0

<::::> x2 -2x+1+ 1 > 0 <::::> (x -1)2 + 1> 0. I.

2

Hin nhie (x - 1)

+ 1 > 0 v6i m9i *x* nen ta c6 bat dftng thtrc can chll'ng minh.0

**Vi d** 3.Chl1ng minh rang neu *a, b,* c la de) dai ba qmh cua *mqt* tam gic thl

*(b* + *c* - a)( + *a* - *b)(a* + *b* - c) < *abc.*

* *Gidi.* Ta c6 cac bat dng thuc hin nhien sau :



2 · 2 2 . .

*a* > *a* -\_ .<*b* -c) = (a ....:. *b* + *c*)(a +.*b* - c)

2 2 2 .

*b* ?. *b* -(c -a) = (b - *c* + *a)( b* + *c* - *a).*

2 2 2

c *?.c -( a - b)* = (c -a + b)(c + a - b).

n.o *a, b,* c la d(> dai ba cnh cua m<)t tam giac nen tat .ca cac ve cua cac bat I

'

dang thuc tren deu duang. Nhan cac ve tUO'llg trng ciia ba bat dng thtrc tren, ta dugc

1

2 2 2 2 2 2

I

*a* ?. (b + *c* -*a)* c *+ a* - *b (a + b* - c . ·

*b* c ( *)* )

Lay can bc hai ciia hai ve,ta dugc bat dang thuc can chfuig minh.

l

0

I

i

I

2• . **Bat d !lg.thuc.v gia tr! tuyt doi** .

. .

Tudjnh nghia gia tr!tuy¢t d6i, ta suy ra cac tfnh chat sau dt\y. .

-lal *a* !al voi mqi *a* e 11t .

Ix!.< *a 6 -a* <*x <a* (voi *a >* 0).

Ix!> *a x < -a* hoc *x > a* (voi *a >* 0):

105

. , .

Scanhed by CamScanner

Sau dAy la hai b4t dfuig thuc quan trc;mg khac ve gia ttj tuy¢t doi (vi€t ducJi

dg b4t dng thuc kep).

lal - *l bl* la + bl !al + lbl (vm mQi *a, b* E IR). Ta chll'ng miilh bat dg thuc I*a+ b* I I*a* I + I*b* I·Tht vy

la+bl < lal + lbl *(a + b 2*

¢:> *)*

a2 + *2 labl\_* + *2*

2 *2*

*b*

¢:> *a2* +*2ab* +*b2* a

+ 2 labl +*b*

<=> *ab* labl.

B4t dg thuc cuoi cimg lu6n dung nen ta co bat dAng thuc can chll'ng minh.

lH1l *Sii dvng bat d!Jng th(lc vU'a chung minh va d!Jng thac* I*a* I = I*a + b* + (-b)I *dd*

*chlmg minh bat dlmg th(lc* Ia.I-I*b* I I*a+b* I·

* 1. Bat ding thuc gifra trung binh cc)ng va trung blnh nhan<l)
     1. Doi vm haiso kh6ng am

Ta da b. , *a +b* I' trung b'

CQA ng cu• a hat• -SOA, *a* va *b.* Kh1•

*a* va *b* kh6ng ArTI

1et a mh

(lll•

thi *J;;b* gQi la trung blnh nhAn cua chung. Ta c6 dinh Iisau day.

DJNH Li

- vm ffiQi *a* ;?: 0, *b* ;?: 0 ta c6

*a+ b* r;-

2;?: *vab.*

Ding thuc xay ra khi va chi khi *a = b.*

N6i each khac, *trung binh c<)ng CUa hai so'khong am /(Jn h(Jn hoiic bang tr"'1g bm' h nha*A*n cu'Ia chu*,*ng. Trung bm' h c<)ng cua hai so'khong am bdn*. *g rrung bi11h nhan cua chung khi va chi khi hai so'd6 b&ng nhau.*

( J) Ngttm ta con gQi Ia bat dAng thuc CO-si (Augustin-Louis Cauchy, 1789 - 1857).

106

Scanned by camscanner

*Chltng minh.* Vai *a* 0, *b* 0, ta c6 .



· . *a+ b l."b"*

1 , . *" !.""* 1 *Fa . . Jb*

-. -- *vao* = -*(a + b* -2vab) = .....: ( *a* -

.

2 . . 2 . 2 · '

Do do

*b)2* > O

- .

.

*a +b" !.""*

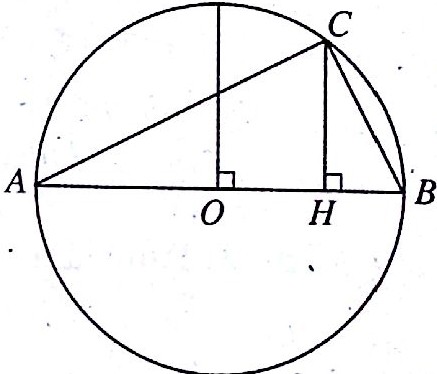
- -2: *vab .*

. / 2

.Dfuig thuc xay ra khi va chi khi (*.F;* - *Jb*)2 = 0, tuc fa *a* = *b.*. D

IH21 *Trang. 17/nh 4.1, cho AH* = *a, BH* = *b. Hay tinh D .*

*cc aoc;1n OD va HC theo a va b. Tit <16 suy ra bat Clang th(tc gi[la trung binh c(mg va trung binh nhan*



*cua a va b.* .

Vi d\f 4. Chll'ng minh rang neu *a, b, c* la ba so

duong bat Id thi

*a+ b* + *b+c* + *c+a* 6.

*c .a b*

*Gidi.* Ta c6

*a+b b+c c+a a b b c c a*

-+ -- + -.- = - + - + - + -+ - + -

*c a b c c · a a b b*

= ( +!)+(+ ) + (>:)

2: 2 v*J*"h*a*·-*b*; *+2tc bc +2ta .ac*

= 6.

·

*Hinh 4.1*

D

HEA

QUA'

*Neu hai so' dudng thay d6i 17:hung c6 to'ng* ·*khong d6i thi tich cua chung /(Jn nhat khi va chl khi hai so'd6 bdng nhau.*

*Neu hai so'dudng thay "d6i nhung c6 tich khong d6i thi t6ng cua*

.· *chung nho.nhdt khi va chi khi hai st/ d6 bdn.g nhau.*

*Chung minh.* Gia sir hai so ducmg *x* va *Y* co t6ng *x* + *Y* = *S* khong d6i. Khi d6,

2

2*S* ::::

.

I

*x* +*y*

> *"cxy:*

nen *.xy* 4S

*Dz* - th'

ang uC

. 2

*5*



xa'y ra

kh. ' va' ch1' khi *x* = *y*. *.*.

.Do d6,tfch *.xy* dt gia° ttjIOn nha't bilng

*4* khi va chi khi *x* =*y.* .





. . - *P* khOng d6i. Khi d6

co

Ohl sfr hai *s6* chrcmg .r vi\ *y* t fch *-'Y* -

:0:: *p = JP* nen *x +y 2: iJP.*

*')* ., . .

- · d6 *11.* g *x* + *y* dt gm tq nho nhat

D;".\ng t httc xa,,y ra kI11. V<:-1..

cIu, kl1,·.\.· - .'•

Do ' tun

bung 2 *fP* khi vi\ chi khi *x* = *y .*

UNG DUNG

.

*Trong*

*cd c<k ltinh clnr*

*J A* • ,*cilnR chu vi, hinh vuong c6 dien*

*'1*

*tdl*

*tich 1<!11 nhal.*

*n 11 co* ·" ·

* - *J* ,. • , • *'no dien tich, hinh vu6ng c6*

*tat*

*Tro11g cd cac hinh chu n ·1{1t co cu o* .· .

*c/111* ,.; *11/1<) 11/1(/ f.*

. . 3 ,. 0

V1,d /!' .nm g·ia, tq• nIiut nIi"r1t cu• a ha'm *s6 Jrt*\x) - *x* + *x*

vat *x* > .

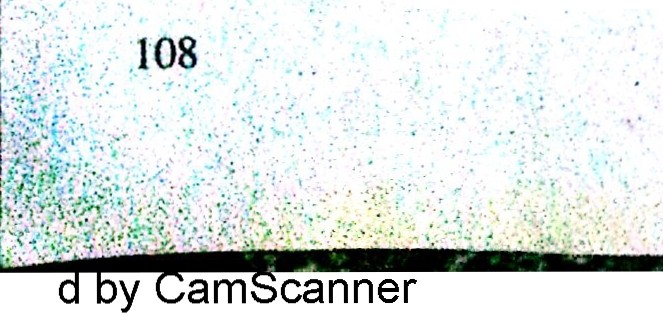
*Gidi.* Do .r > 0 nen t.a c6 *f( x)* = *x* + *x*

2: 2 *Qx·;'* = 2J3. va

.f(x) = *2J3* <=> *x* =. <=> *x* = *J3 .*

*x*

vy gia tr! nho nhat d1a ham s6/(x) = *x* + v6'i *x* > O la/(*J3)* = *2.J3.*



. *x* .

b) D6ivm ba so khong am

Ta da biet·.

*a* +*b* + *c* la' tnmg 3

b'mh c¢ng cu'a

ba so' *a, b*, *c.* Ta g91·

*v3ra;ob-\_c*

\a'

trung blnh nhan cua ba s6 d6. NgtrO'i ta ci:ing chg minh duqc ket qua tuong

tr d!nh Iitren cho truO'ng hqp ba s6 kh6ng am.

V0-i m9i *a 2* 0, *b 2* 0, *c .2* 0, ta c6

*a + b+ c*

3

*2 v*3,*a*-*h*;*c*­*.*

Dfing thuc xay ra khi va chi khi *a = b* == *c.*

N6i each khac, *trwig hinh c<)ng Clta ba so'kh6ng dm l6'n hon hoiic bang fJ'llll*

*hinh nhdn c1.ia chilng. Trung hi11h c{lng ciia ha so'khong .dm ba1g trwg bi11/*

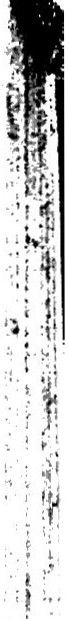
,. *nflci 1 cua cltlng .khi va chi khi ba so'd6 hang nhau.* "

*:* .

Scanne



*Vi* d1;1 6.Chll'ng niinh rAng neu *a, b, c* Ia ba *s6* du<111g thl



*(a + b* + c) (.!.+.!.+.!.J 2'.: 9. ·

*a b c*

Khi nao xay ra ding thuc ?

*Gidi.* Vl *·a, b, c* Ia ba *s6* du<111g nen

*a* + *b* + *c* 2'.: 3 (ding thuc xay ra khi va chi khi *a = b =* c) va

!' ' '

*:::* ·.,

I+-l+-l 2'.: 3 1 (dA., ng thu,c xa.,y ra kh1.va,chi kh

1. I

= l*=*-I)· r •

*a b c a a b c*

Do d6 *(a + b* + c) (.!.+.!.+.!.J 2'.:.3 .33/i *=* 9.

v-;;;;;;

*a b c*

*a =·b =c*

Dng thuc xay ra khi va chi khi J\_ = .!\_ = .!\_.

*{*

I . ..

i .

·

.



. *a* .

*b c*.

Vy dg thuc xay ra khi va ·chi khi *a = b* = *c.* 0

IH3I *Phat bieu ket qua tLIOhg ti/ h qua ophan a) cho trlfiJng hqp ba s6 dLJOng.*

**cau hoi va bai ta**.**p**

1. Cht'.mg mm. h ra,ng, ne·'u *a* > *b*

va'

*ab* > 0

th1' I

<-I .

*a b*

2. Chll'ng minh rang.mra chu vi cua m¢t tam giac Ian hem d¢ dhl m6i ccua tam

giac d6.

3, Cht.'rng minh rang *a2* + *b2* + *c2 ab* + *be* + *Ca* yffi m9i *S6* t\!C *a, b, C.*

Dg thuc xay ra khi va ch\_i khi *a* = *b* = *c.*

1. Hay so sanh cac ket qua sau day :
   1. *.J20i50* + fi.005 va .J20o2 + Ji.003 (kh6ng dung bang *s6* hoc may tinh).; b) *.Ja* + 2 + *Ja* + 4 va *-Fa* + *.Ja* + 6 *(a* 0).



*.f* . .

109

Scanned by Camscanner



|  |  |  |
| --- | --- | --- |
|  | . . 1 1 | 4 |
| *5.* | Chtlng minh rang, n€u *a* > 0 va *b* > 0 thl -;; + *b* | *a* + *b* |

**6**· Chu'ng m1·nh

ra·ng, n,u *a*

o va' *b* >- O thl a3 + *b3 ab(a* + *b).* Ding thi"t'.\.;

.. xay ra khi nao ?

7. a) Chung minh rAng *a2 + ab + b2* O vm m9i *s6* thl!c *a, b.*

b) Chti'ng inh rAag v(ri hai *s6* thl:fC *a,b* tuy y, ta c6 *a*4 + *b*

4

> *a b* + *ab3*

*3*

**8.** Cl\_ill'ng minh rAng, n€u *a, b, c* Ia d¢ dai cac c4µ1h cua m(>t tam giac thl

*a2* + *b2* + *c2* < *2(ab + be + ca).*

9. Chll'ng minh rang, n€u *a* 0 va *b* 0 thi

*2 3 3*

*a* + *b* x *a2* + *b*

< *a* + *b*

2 2 - 2

1. a) Chli'ng minh rfulg, neu *x y* 0 thi *x y*

*I + x I + y*

b) Chll'ng minh rang d6i vm hai *s6* tuy *y a,b,* ta c6 *l a -bl* < !al + lbl

1 *+ l a-bi* I + lal I *+ l bl*

1. Chll'ng minh rfulg :·



* 1. Neu *a, b* la hai *s6* cung dau thi *a* + *b* > 2 ·

*b a -* '

*I ..*

b) Neu *a, b* la hai *s6* trai dau thi *a + b* < -2

*b a -* ·

1. nm gia tri Ian nhat va gia tri nho nhat cua b>-- SA /( ) ( ) *Ji.*

. . ruu u *x* = x + 3)(5-x vu1

*-3 < x 5.*

'13. nm gia tri nho nhat cua ham *s6 f (x)* -*x* + ·2 ,.

· · ,

- -vm x > I

* 1. ·

...

110

I

.j

Scanned by CamScanner

,

* + 1. Bit dAng th(tc Bu-nhl-a-c6p-xkt d6t vat hal cp s6 th

*val* hai P s6 thl,lc (a,b) va (x,y) ta c6

*(ax + hy)2* 5 *(a2* + *b2)(x2* + /).

£)ng thCtc xay ra khi va chikhi*ay = bx.*

*Q.htlnO mlnh.*

oA dang ch(lng minh dang th(Jc sau :

*2*

*(ax* + *by)*

+ *(ay* - *bx 2*

= (a2 + b2)(x2 + /).

Mt khac, do *(ay* - *bx) 2* o *nlm*

*)*

*2*

*·(ax + by)*

+ *(ay* - *bx 2*

(ax + *by)2 .*

Tlt d6 suy ra bt dang th(/c djn ch(tng minh.

*)*

E>Ang thCtc xay ra khi va chikhi *ay* - *bx =* o. *t(/c* 1a *ay = bx.*

*Chu y.* Khi*xy t=* O, dieu kin *ay = bx* c6n dl!gc viet dl!l1i dfiing *a* = *!:.*

*x y*

2. Bat ding th(tc Bu-nhla-c6p-xkl doivdi hai bQ ba soth\fc

C6 th6 ch(tng minh ket qua sau :

.' \ ·.

'I '

 \ ·...



Nu *b1 b2 b3 f.*O th! dang th(/c xay ra khiva chikhi = *a2* = .

b, *b2* b3

Vi d'if. ChCtng minh rng n6u *a2* + 21/ + *9 2*

*c*

= 3 thi *a* + *2b* + *9c* :s6.

2

*Gisi.* Ta c6 (a + 2b + 9c)2 *= (a.*I + *J2 b.J2* + 3c.3)

2 2 2 2 2 *2* 2

s[a

+ (../2 *b)2* + (3c)

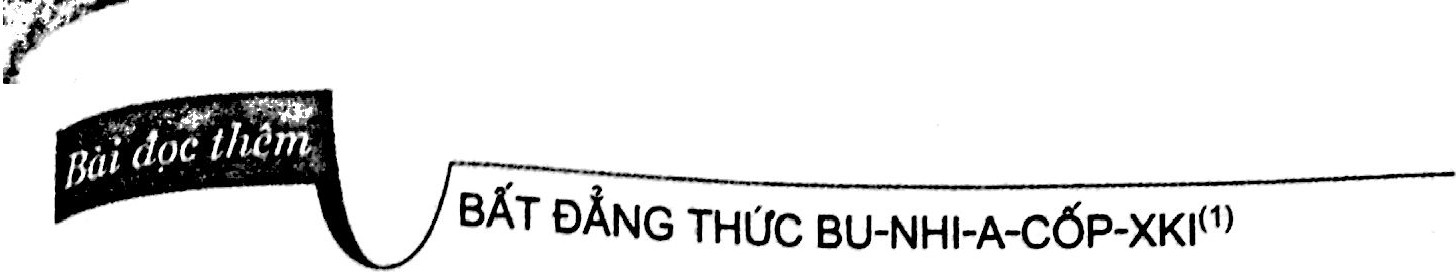
J l l

+ (fi.)

+ 3 ) = 12 ca2 + *2b*

+ 9c ) = 36.

Vi vy *a* + 2b + *9c* 6. D



...... ---

1. Yiktor Yakovlevich Bunyakovsky ( t 804 - 1889), nha toan hqc\_Nga.

lll

·;, . '

.,.

Scanned by Camscanner

* + **Luvtn lP-**

**14.** Cht1ng minh rg neu *a, b,* ·la ba s6 duang thi

*4 b4 c4*

*0*

-+ -*+ -?.3abc.*

*b c a*

**15.** M¢t khach hang den m<)t cita hang ban hoa qua mua 2 kg cam da yeu c'au

c.An hai Jan. Lan dau, ngucri ban hang dt qua .cAn 1 kg Ien dia can hen phai va dt cam ltn dia cAn ben trai cho den khi cAn thang brtng va Mn sau, dt qua cAn I kg Ien di cAn ben trai va dt cam Ien dia can ben phai cho den khi

ca.n thang biing. Neu cai cAn dia d6 kh6ng chfnh xac (do hai canh tay don dai, ngAn khac nhau) nhtrng qua cAn Ia dung lkg thi khach hang c6 mua duqc dung 2 kg cam hay kh6ng ? Vl sao ?

**16.** Cht1ng minh ding v& mQi *s6* nguyen ducmg *n,* ta c6 :

I . I I I I

a) -+ -+ -+ ... + < l ;

1.2 2.3 3.4 *n( n* + 1)

*Huo*,*ng*

*da;:n.* V"1et

1 = 1 - l · l

= 1 - -

1.2 2 ' 2.3 2 3 ' ...

I I I 1

b) 2 + 2 + 2 + ...+ 2 < 2·

1 2 3 *n*

**17.** lim gia tr!Ian nha:t va gia tr! nho nha't cua bieu thuc

19. QnIDg minh ning ne'u *a,b, c,d* 13 b6n *s6* kh6ng fun thl ( a + *b : .c* + *d* r-;,*abed .*

1. Cht1ng minh rilng :
   1. Neu *x 2* + *y 2* = 1 thl Ix +*y* I *.J2* ; b) Neu *4x* - *3y* = 15 thl .r2 + *"2* 2 9.

t 12

Scanned by CamScanner

\_\_,---.... ---

*§ 2* EJI CTJONG vt BAT PHTJCING TRiNH



# f!!BiJ"

1.Khai nim bAt phuang trlnh mc)t An

\

..

DJNH NGHIA

*Cho hai ham st/ y =f( x) va y* = *g( x) c6 t(lp xac djnh ldn ht(lt la '-J'i1*

*va Y'g. Dijt* 1= *<J'i1n g)g·*

*M¢nh de' cha bien c6 m<)t trong cac dqng f( x)* < *J:(X ),*

*f( x)* > *g(x),f(x ) g(x), f(x)* 2: *g(x) dur;c gqi la bat phttdng trinh mt)t an* ;*x gqi la an st/ (hay tin) va* qi *gqi la p xac dfnh cua*

I *bar phuong trinh d6.*

l

*So'Xo* E l *gqi la mf)t nghim cua btft phucmg trinh f( x)* < *g(x)*

*nluf( x0)* < *g(x0) ta m¢nh de' dung.*



Khai ni¢m "nghi¢m" cling duqc djnh nghia nrong n,.r cho cac bt phuong trlnh dg

*f(x)* > *g(x),f(x ) g(x)* va *f( x)* 2: *g(x).*

*Gidi mt)t bilt phttdng trinh* la tlm tAt ca cac nghi¢m (hay t1m *t(lp nhiefm)* cua

bat phucmg trlnh d6.

CHO Y

Trang thlfC hanh, ta khong can vit ro tp xac djnh *(]'* cua bat phucmg trlnh ma chi can nu di u ki¢n d *x* E *9'.* Diu ki¢n d6 g9i

la *dieu ki¢n xac djnh cua bat phum1g trinh,* gQi tAt la *diell kin CUa*

*bat phu<111g trinh.*

IH11 *Bidu dien tp nghim ctia m6i bat pht.JrJng trinh sau boi cac kl hiu khoang*

*hoc don :*

*a) -0,5x* > 2 ;

8·Oi S6' 10·NC·A

Scanned by Camscanner

*b)* lxl .$ 1.

I13



*>*•*'*.

'·

..\_.llmll... .. u

, d' *R* r) < *g(x).* D6i VOi Cac bat Dtt6i day,chung ta chi n6i t&i ba't phttcmg tnnh &;ing; v - g c6 cac ket . phttcmg trlnh d&;lng *jf x)* > *g(x), j{ x) g(x)* va *j{ x)* 2! *g(x),* ta cun qua

tttcmg *tt!.*

2. Bat phuong trlnh tuang dttoog

DNH NGHlA

' .• *) d* '(Ji *ooi la 111ung duung ne/,*

*Hai hat pl111lfn8 trinh (cunf? an w. c* ,.,.

I *chung c6 nlng t(lp nghi¢m.*

*N et1f 1(x)* < .1:1(x) */11(/ng ducmg vilif2<x )* < *g1(x) thi ta ,,ie!*

*f 1 (X )* < *g 1(X )* ¢:::> *f2( X )* < g2(.r).

IH21 *Cac khlmg c1fnh sau dy dung hay sai ? Vi sao ?*

*a) x* + *J x* -2 > *J x* -2 <::> *x* > 0 ;

*b)* ( )1 I *<::::> x* - I I.

CHO Y

Khi mu6n nhan m.m h hai bat phttcmg tnnh c6 cung t[)p xac djnh lJ" (hay c6 cung dieu k i¢n xac djnh ma ta ciing k f hi¢u la lJ ) va tucmg dttcmg v<Ji nhau, ta n6i :

- Hai bat phucmg trlnh tucmg ducmg tren CJ , hoc

- v<Ji dieu ki¢n J\ hai bat phucmg trlnh la tu<111g du<111g vffi nhau.

Vi du I.V6i diu k ien *x* > 2, ta c6

1

> I I > *x* - 2. 0

· · *x - 2*

3. Bien doi tu<111g duong cac bat phuang trlnh

Ciing nhtr v6i phucmg trlnh, *(J* day chung ta quan tArtl den cac phep bien d6i khong lam thay d6i tp nghi¢m cua bat phucmg trlnh. Ta g9i chung la cac phep *biln dti'i tu<mg dullng. Phep hiln do·i r11011g du<mg bien nu)r hat plurm 1 1ri11/I*

*thimh m¢t hdt ph11m1g trinh rum1g ducmg vm no.* Chng hC;ln , vie.c tht.rc hie.n cac

phep bin d6i d6ng nha't *b* m6i v cua m¢t bat phucmg trlnh va gifr nguyen t P xac d!nh cua n6 la m¢t phep bi n d6i tucmg ducmg.

114

1. Di*S6* 10 -NC·S

Scanned by CamScanner

--- ,.. *-:::a....-. >* .



*D* .day la dnh If v m¢t *s6* phep bin d6i tuang duang thuang dung. Qic ham

*s6* not trong d!nh Iinay deu duqc cho OOi biu thl'tc.

DJNH Lf

*Cho bdt phumzg trlnhf(x)* < *g(x) co t(lp xac djnh* 9),*y* = *h(x)*

*la m9t ham so'xac djnh tren J.*

*Khi do, tren* y ' *bat phu011g trinhf(x)* < *g(x) IUO'ng dudng viii*

*moi bdt phu{lflg trinh :*

I)*f(x)* + *h(x)* < *g(x)* + *h(x)* ;

2)*f{x)h(x)* < *g(x)h(x ) nlu h(x)* > 0 *vfli m9i x E* J) ;

3)*f(x)h(x )* > *g(x)h(x ) ne'u lz(x)* < 0 *viii m9i x E* g).

*Chtlng minh.* Sau day, ta chi chtlng minh ket Ju 3). Cac ket Iu khac cling duqc chll'ng minh tuang ttJ.

Neu *Xo·* thu¢c *y* thl *f (xo ), g( xo)* va *h(xo )* la cac gia ttj xac djnh bang *s6,*

han nii'a, vi *h(x)* luOn Am nn *h( x0 )* < 0. Do d6, ap dl;lng tfnh chat cua bat ding thl'tc *s6,* ta c6

*f (x0 )* < *g( x0)* <:::> *f (x0)h( x0)* > *g( x0 )h( x0 ).*

Tir d6 suy ra rAng hai bat phuang trlnh c6 cung tp nghi¢m , nghla Ia chung tu<111g dttcmg vai nhau. o

Vf d 2

* 1. t phttcmg trlnh /;> -2 tuang duang vai bat phuang trlnh

/;-/;*>-2 - Fx.*

* 1. Mt phircmg trlnh *x* > -2 khOng tuang duang vm bat phuang trlnh

*x -/;* > -2 -..Jx. . D

IH3l *Chifn.g minh cac khdng djnh trong vi di! 2.*

IH4j *Cac khdng dinh sau dAy dung hay sai ? Visao ?*

a1 x . I

+ -

< 1 + I

x < I;

*b'' x( x -1)* < 2

*/* -

·< 2

*-* ·

*x*

*I x x*.

-

x - J

J 15

Scanned by CamScanner



H QuA

*Cho bat plut<mg trinh J(x)* < *g( x ) c6 u;lp xac djnh* ·

*I ) Quy r&c ndng /en luSi thita h<Jc ba*

*ftx)* < *g(x)* [Jtx)]3 < *[g(x)] 3.*

*2) Quy tdc nang /en lu)i rhila b<jc hai*

*N el1*f{x) *va* g(x) *khong am v6i m9i x thu<)c* 1*tlzi*

Rx) < *g(x)* [ftx) 2 < L (x) 2

] ] ·

Tuong tv.ta ciing c6 quy tac nang ten luy thira bc le va rnlng len luy thira bc chan.

IHs] *Giai bat phl./Ong trinh sau ay (bng each binh pht!Ong hai ve), giai thich ro* cac

*phep bien c16i tl./Ong c11.10ng da tht/C hin :*

Ix + 1 1 Ix !.

**cau hol v bl tip**

•

1. M¢t bn l p lun ntur sau : Do hai ve cua ba't phtrcmg trlnh *.Jx* - I < *I x* I

luon kho.n., g am nen blnh phtrcmg hai vf,, ta dtrqc bat phtrong trlnh ttrcmg duang

.r - I <.r.I;.Theo em, lp lun tre.n c6 dung khong ? Vi sao ?

1. nm dieu kin xac d!nh r6i suy ra tp nghi¢m cua m6i bat phtrcmg tr1nh sau :
   1. E> *h ;*

t

c) *x* + 2 + --

.r - 3 *x - 3*

b) *.Jx* - 3 < I + *.Jx* - 3

d) *x* < 2 .

*.Jx* - 2 .J.r - 2

1. Trong hai bat phucmg trlnh sau dAy. bat phtrcmg trlnh nao tuong dtrong v6i ba't

phuong trlnh 2\·- I 0 :

*2x* - 1 + va 2r - 1 - >

..r - 3 .r - 3 *x + 3* - *x + 3*

1. Trong b6n cp bat phttcmg trlnh sau day, hay ch9n ra cac cp bat. phtrcmg trlnh

ttrcmg duong (neu c6) :

I16

a) *x* - 2 > 0 va *x2(x* - 2) < 0 ;

c) *x* - 2 0 va *x2(x* - 2) 0 :

b) *x* - 2 < 0 va *.x2(x* - 2) > 0 :

d) *x* - 2 0 va *x 2(x* - 2) 0.

Scanned b CamScanner

*J*



·***':J*** BA P ONG TRiNH vA,i-m sAT PHVONG

*§* TJUNH BC NHAT M\>T AN

. ..... ... -

- ,,. .,:. ? .

Tru6'c day, chung ta da lam quen v6'i *bat phuO'ng trinh bt;lc nhat mt)t an.* D6 la bat phuong trlnh c6 m¢t trong cac dC;Ulg *ax* + *b* < O, *ax b* 0, *ax* + *b* > 0, *ax + b* 0,trong d6 *a* va *b* la hai so cho truoc vm *a i=* o,*x* Ia *tin.*

IH1J *Cho bat phl.ICJng trinh mx m(m* + 1).

* 1. *Giai bat.phl.ICJng trinh voi m* = 2.

*b) Giai bat ph1.1CJng trinh voi m* = *-.Ji..*

Nhu vy' neu *a* va *b* Ia nhii'ng bieu thuc chua tham so th1 tp nghi¢m cua bat phuong trlnh pht;1 thu(>c vao tham *s6* d6. Vi¢c tlm tp nghi¢m cua m(>t bat

phuong trlnh tuy theo cac gia tri cua tham so g9i Ia *gidi va bi¢n lut;ln* bat

phtiong trlnh d6. ·

Dum day' chung ta chu yeu n6i ve each giai va bi¢n lu bat phuong trlnh dC;tng *ax + b* < 0. Doi vm cac bat phuong trlnh dg con l<;ti, each giru cilng tuong *ttf .*

**1. Giai va bin lun bat phuong trlnh dng** *ax* + *b* < **0**

Ket qua gia{va bi¢n lun bat phuong trlnh

*ax +b<O* (1)

dliqc neu trong bang sau day.

I ) Neu *a* > O tl (I) e> *x* < - .V Y tp nghi¢m cUa (I) Iii S= (-oo ; - J

2) Neu *a* < O th!( I ) <=> *x* > - *b* . V y tp nghi¢m cua (1) la *S* = (- ;+oo).

*a* . *a*

3) Neu *a =* O thl ( I) <=> *Ox <-b.* Do do :

-Bat phuong trlnh ( 1) v6 nghi¢m (S = 0) neu *b* 0 ;

\_ Bat phuong trlnh (1) nghi¢m dung vm m9i *x* (S= JR) neu *b* < 0.

lI .

1 17

Scanned by Camscanner

CHO Y

Vi¢c biu dien cac tp ngbim trn tfl;lC *s6* se rt c6 ich sau nay.

Chfuig hC;in, phan kh6ng b!gch *a* tren hlnh 4.2 biu din tp nghi¢m cua ( I ) vm *a >* 0.

,,..,.

...,..

***rrrrr11111***

*b*

-

*a*

1 -

*Hinh 42*

.1 Vi d l. Giai va bi¢n lu bat phuang trlnh



*nu +* I*> x + m2•* (2)

*Gidi.* Bat phuong trlnh (2) tuang duang voi

Ta co

*(m* - I).x > *m*2 - I. ·

(3)

I) Neu *m* > Ithi *m* - i> 0 nen (3) <:::> *x* > *m* -1 <:::> *x* > *m* + l.

2

· m-1

2

2) Neu *in <* Ithl *m* - 1 < O nen (3) <:::> *x* < *m* -l <:::> *x* < *m* + 1.

m-1

3) Neu *m* = I thi bat phuang trlnh tro thanh *Ox >*O nen n6 v6 nghi¢m.

*Ket lutjn :* - Neu *m* > 1 thl tp nghi¢m cua (2) la *S* = *(m* + 1 ; +oo).

- Neu *m* < Ithi tp nghim cua (2) la *S* = (-oo ; *m* + 1).

- Neu *m* = 1 thi tp nghi¢m cua (2) la *S* = 0. ·

D

IH21 *Titket qua tren, hay suy ra tp nghim ciia bat phLJOng trinh*

*ntx +* l>*\_ x + m*2*.*

1 18

Vi d 2. Giai va bi¢n lun bat phuang trlnh

*2mx x* + 4m - 3.

*Gidi.* Bat phtrcmg trlnh (4) tuong duong vm

*(2m* - I )x 4m - 3.

(4)

(5)



, I , *4m -3*

l) Nu *m* > - th1 *2m* - *1* > 0 nen (5) <=:> *x* ·

2 *2m - l*

,

*m* <

2) Neu

I .. *2m* - I < O nen (5) <=:> *x*

*4m -3* ·

2 *2m - l*

- th1

3) Nc!u *m* = thl(5) tr& thilnh *Ox ;,,* -1, bOi vy n6 nghi¢m dUng v6'im9i .r.

*Ke*\_*t*, */ Uf1*.*11 :* - Ne,u *m* >-I th!tp nghi¢m cu.a (4) *la*.. *S* = *[ 4m* -3 ; +oo J.

2 *2m -I.*

- Ne..u *m* <-J

th' tp ngh1. ¢m cu• a (4) la' S = (-oo ; 4

-3J.

1 m

2 *2m -1*

- Neu *m* = \_2.!\_ thi ta.p. nghie.m cua (4) la *S* = IR . D

**2. Giai h bat phuong trlnh bc nhat m(>t an**

Tuong 11! nhtt h¢ phttong trlnh, tp nghi¢m cua m¢t h bat phucmg trlnh Ia

giao cua tat ca cac tp nghi¢m cua cac bat phttong trlnh trong h¢.

Do do,

*Mu6!1 gidi h¢ bat plu tdng trinh m<)t an. ta gidi titng bat phuang*

*trinh cua h¢ r6i lay giao cua·cac t(lp nghi¢m thu duqc.*

**Vi d1;1** 3.Giai h bat phuong trlnh

*3x* - 5 0

(6)

(I)

*2x* + 3 0

*1x* + I > 0.

(7)

(8)

*Gidi.* Giai Ian lttqt ttrng bat phttang tr1nh cua h¢, ta dttqc :

Tp nghi mcUa (6) Iii S1 =(-«) ; ] ; Tpnghi¢m cUa (7) Iii 52 =[- ;+oo) ; Tp nghi¢m cua (8) Ia S3 = (-1 ;+oo).

1 19

Scannea by CamScanner

....



Vy tp nghi¢m cUa h¢ bat ph11ang trinh dii cho 1 ]

5

S= S1 *n* S2 *n* S3 = (-1 • 3 · 0

Ta ciing c6 the trlnh bay lm gi.ai vi d9 3 nhu sau :

5

*x -*

3

( ) <=> *x* - <=> -1 <*x .*

2 3

x > -1

Tp nghi¢m cua h¢ bat phuang trlnh da cho la

S=(-1; 1 D

CHO Y

*Dt* de xac d!nh tp nghi¢m *S,* ta biu din cac p ngqi¢m tren tn.;1c

so bang each gch di cac diem (phful) kh<>ng thu(>c tp nghim cua

titng bat phuang trlnh trong h¢, phful con li se biu dien tp nghim c tlm (h.4.3).

5



*-1-* -1



2 3

*Hinh 4.3*

lH31 Tim *cac gia trj cua x dd d6ng thoi xay ra hai dang th(lc :*

I3x + 2 1 = *3x* + 2 *va* 12x - 5 I= 5 -2x.

*Huimg da11. 1A l = A* <=> A O va 1B 1 = -B <:::> B O.

Vi d\I 4.Vc1i gia ttj nao cua *m* thl h¢ bat phuang trlnh sau c6 nghi¢m ?

x + *m* < 0

{*-x* + 3 < 0. (9)

(10)

*Gidi.* Ta c6 (9) <=> *x -m.* Tp nghi¢m cua (9) la (--oo ; -m].

(10) <=> *x* > 3. Tp nghi¢m cua (10) la (3 ;+oo).

va·y tA·p nghie·m cua he· la *s* = (-oo ; -m] n (3 .' +,.....,,.....,.) ·HA

co,.

ngh'1em

k,h'1 v-.

120

chi khi *S \** 0,ttrc fa 3 < *-m* hay *m* < -3. . ·

0

scnnea oy CamScanne(

**cau hol va bal tap**

•

*25.* Giai cac bat phuC111g trlnh :

*x + 2*

a) - *x* + I > *x* + 3 ;

b) *3x* + 5 \_ I

*x* + 2 + *x* ;

3

c) (1 - *Ji.*)x < 3 - 2Ji. ;

26. Giai·va bi¢n lu cac bat phuC111g trinh :

2 3

d) (x + *J3*)2 (x - *J3*)2 + 2.

a) *m(x* - m) *x* - I ; b) *mx* + 6 > *2x* + *3m* ;

c) *(x* + l )k + *x* < *3.x* + 4 ; d) (a + l)x *+ a +* 3 *4x* + 1.

27.. Giai cac h bat phUClllg trlnh :

*5x* - 2 > *4x* + 5

a) *{ 5x* - 4 < *x* + 2 ;

*2x* + 1 > *3x* + 4

b) *{ 5x* + 3 *Bx* - 9.



1. Giai va bi¢n lun cac bat phUClllg trlnh :

a) *m(x* - m) > 2(4 -x) ;

1. *3x* + *m*2

*m(x* + 3) ;

c) *k(x* - 1) + *4x* 5 ;

1. Giai cac h¢ bat phucmg trlnh :

*5x* + 2 4 \_ *x*

* 1. *b(x* - I ) 2 -*x.*

(1 - *x )2* > 5 + *3x* + *x2*

* + 1. 3

*6*

*- 5x*

--< *3x +* I·

13 '

*4x - 5*

b)

{(*x* + 2 < *x3*

f

+ *6x2*

- 7*x* - 5 ;

. *< x + 3*

c) 7

*3x* + 8 > *2x* - 5 ;

4

*x* - I *2x* - 3

d) *3.x* < *x* + 5

5 - 3.r

-- *x - 3.*

2

1. Tim cac gia tr!cua *m* d m6i h¢ bat phuang trinh sau c6 nghi¢m :

a) {3x - 2 > *-4x* + 5 b) *{x* -2 0

*3x* + *m* + 2 < 0 ; *m* + *x* > 1.

1. lim cac gia tr!ciia *m* de m6i h¢ bat phuang trlnh sau vfl nghi¢m :

a) {2.x + 7 < 8.x - I b) {(*x* - 3)2 *x2* + 7*x* + 1

*-2x* + *m* + 5 0 ; *2m* - *5x* 8.

121

by CamScanner



tdJ:...· ....--DAA'u cu'A NHI. TH=-=\_c\_B\_:\_\_c\_N\_H\_A\_'T

* 1. Nhi thuc bc nhat va dau cua no

Nhieu bai toan dAn den viec xet xem mQl bieu thuc*f( x)* da cho nh n gia tri am

a

(hoc ducmg) v6i nhftng

. ·

gia tq

nao

·

cua

1. T
   * ·*t*

1 v1 .

g9 c

lam d6 la *xet dal,1 c1ia*

*b1.e*.•*u thl*,*lC* j°j( ) DUC11. day, ta se-

t1m h.,:> ve,: nh1·. th•'"c bA'?-c nhat va dau cua n6.

*p X .*

IeU u

a) Nhithuc bc nhat

D!NH NGHlA

*Nh! thfrc bt!c nhtft (doz vmx).liJ bilu tht1c dqng ax + b, trong d6 a ra*

.\ I

*b liJ hai so'cho tru6'c viti a =;:.* 0.

Ta da biet, phucmg trinh *ax* + *b* = O *(a =;:.* 0) c6 m<)t ngtii¢m duy nhat *x0* = -*a* . Nghi¢m d6 ding duqc g9i la *nghi¢m cua nhj thitc bgc nhdt* f(x) = *ax* + *b.* N6 c6 vai tro rat quan tr9ng trong vi¢c xet dau cua nh! thtrc bc nhat f(x).

b) Dau cua nhj thuc bc nhat

Dt *x0* = -*b ,* ta viet nh! thuc bc nhatf(x) *= ax + b* nhu sau

*a*

*/(.x)* = *ax* + *b* = *a*(*x* + ) = *a(x* \_ *xo).*

Khi *x* > *Xo* thl *x* -*Xo* > 0 nen dau cua *a(x* - Xo) trimg vai dau cu'a a.

122

.

Khi *x* < *Xo* thi *x* -*Xo* < 0 nen dau.

cua *a( x* -Xo) tnii vai dau cua *a.*

Scanned by CamScanner

**&L&ASI# W ·\_** .J

# *I A* $ •· *' LL* •••

Tl!d6 ta c6

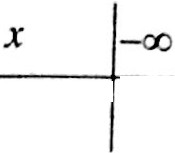
DJNH LI (v dau cua nhj tht1c bc nhat)

*Nhi thitc· hc nhal ftx )* = *ax* + *b cung dtiu vm h¢ stf a khi x /<Jn*

*h011 n1-:hifm Va /raj dt/u v6i h¢ s(f a khi X nhohdll nghi¢m CUa no.*

*y y*

1,5 +oo



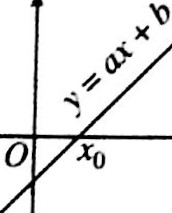
+ 0 *a > O*

Nhj thllc da cho ducmg khi *x* < 1,5

va Am khi *x* > 1,5.

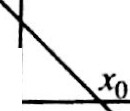
*x 0*

*Hinh 4.4*



*x*

x



*..P*

*a* < 0 *6*

IH1I*Hay g;ci; thich blmg d6 thf (h.4.4) cac ket qua cua dfnh Ii trfm.*

* 1. M(>t so trng d1;1ng
     1. Giai bat phuong trinh tich

Ta xet cac bfft phucmg trinh c6 the dua v m¢t trong cac d g *P(x)* > 0,*P(x)* O,

*P(x)* 0,*P(x)* < 0 v6'i *P(x)* la tfch cua nhitng nhj thll'c bc nhat.

Vi d1,1 1.Giai bat phucmg trlnh

*(x* - *3)(x* + 1)(2 - *3x)* > 0. (I )

*Cach gidi. D* giai bat phucmg trlnh ( I ), ta l p bang xet dau ve trai cua ( I ).

Dt *P(x)* = *(x* - *3)(x* + I )(2 - *3x).*

-Giai phucmg trlnh *P(x) =* 0, ta duqc

(x - 3)(x + I )(2 - *3x)* = 0 <=> *x* = 3 hoc *x* = -1 hoc *x* = ·

123

Scanned by camscanner



2

* - • h h ' t•" t"'ng ·-1 , 3. Ba s6 nay chia

- sap xep cac gia tr!tlm du<JC cua *x* t eo t u au · ' 3

fn:lc *s6* thrum b6n khoang. Ta xac dP1h dau cua *P(x)* tren timg khoang bang each lp bang sau day g9i la *bdng xet dal' cua P(x).* -

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | | | | |
| *x* | -1 -2 3 +<X:l  -00 3  - | | | |
| *x-3* | - I - |  | - 0 + | |
|  |  |
| .\- + I | - 0 + | + | + |
| 2 -3.x | + I + 0 - | | | - |
| *P(x)* | + 0 - 0 + 0 - | | | |

Trong bang xet dau, hang tren cung ghi l i b6n khoang duqc xet cua lf l;!C so, ba hang tiep theo ghi dau cua cac nhan *tit* bc nhat tren m6i khoang (dl;l'a vao djnh *li* ve dau cua nhj thuc bc nhat) ; hang cu6i ghi dau cua *P(x)* tren moi khoang bang each lay "tfch" cua cac dau cung c¢t *(J* ba hang tren.

Dl;l'a vao bang xet dau, ta c6 tp nghi¢m cua bat phuang trlnh (1) la

S= (;-l) u G ; 3} o

* + 1. Giai bat phuung trinh chua An *a* miu

*a* day' ta chi xet cac bat phuang trlnh c6 the dua ve m¢t trong cac dng

*P( x ) P( x ) P( x)* . *P( x)*

-- < 0, - 0 , - 0, -> 0, trong d6 *P(x)* va Q(.,r) ta tfch ciia

*Q( x) Q(x ) Q( x)* Q(x)

nhiing nh! thuc bc nhat. D gia cac bat phuong trinh nhu vy, ta lp bang xet

dA,u cu• a phA*an* th' *P( x)* · Kh' lA'.1P ba'ng xe't dft·u, nh6' a·ng phai ghi tat ca cac

l!C *Q(x)* I

nghi¢m cua hai da thuc *P(x)* va Q(x) len tn;ic *s6.* Trong hang cu6i, ti nhfmg . diem m Q() = O, ta dung kl hi¢u II de chi ti do bat phucmg trinh da cho khong xac d!nh. .

Vi dl,I 2. Giai bat phuong trlnh

-3- <-5-

*x* - 2 - *2x* -1

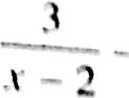
124

Scanned by CamScanner

*r Girii.* Ta c6

I (:.:!)

-- :,, 0



1.r - I

<-- 3( .2.,. - I)-. (· r ')-) .

l-' - :!)( .,.r • I ) . ()

.r t- 7

(.r - )(2.r - I)

B<ing xet dtfu vc' 1n\ i Cl0la (.l) :

0. (.1)

-



.\' -( -7 I 2

*l*

.\' -f 7 - 0

\' - "'\ --·==--1

·I I

<)

!r - I () *I*

Vt.Ir.ii - 0 II I

Tu d6 suy ra tp nghi¢m ctia (2) I *S* = (-"' : 7 1 u ({ : 2 ).

c) Giai phucmg t rinh, bfit pluron t rlnh rlura t•n tron dflu ia t r t uyvt dc'ii

M¢t trong nhfmg crich ci:\i ph tt<tng lrl nh hay h:tt phtrang t rinh ·h t'ta :ln l rong

dau gia *tri* tuyt d6i la . *t't* dt.mg d!nh nghia c1J khtr diiu gi(i I r! t uy . 1 <16i. Ta

.thttemg phai xct phtrcmg 1rl11h hay h t ph tt<'mg 1rlnh t rong nhicu k ho:ing «to:.in, ni.ra khoang) khflc nhau. t rCn do m i hiCu lhfrc nf\rn t ron • dAu gifi t r! 1u y 1 d()i d6u c6 mo.t dclU xac d.inh. Sau c1:ly Ii\ m()I vf dt.1 don gi:ln.

Vi d1;1 3.Giitibtlt phum1g t rl nh

1 2r - 1 1 < 1r + 5. *( 4 )*

*Gidi*

l ) Vm x < .ta c6

(4) ¢=> 1·- 2.r < *3x* + *5* . 5.r -4

I 4

4

> .\' / --·

5

K·t hqp v6i ditu ki¢n .r < 2 . t a dtrc *5* .r 2 V y t p ca<.: nghi m t hoa

man ditu ki¢n dang xCt Iii khming (- : ).

Scanned by camscanner

01..\ \_ - ,ta co



2) V ,. .> I ,

2

(4) *2x* - I < *3x* + 5 *x* > -6.

Ket hrtn vm dieu kien *x* \_!\_ . ta dUCJC *x*



2

';'t' .

1 . Y$y tp cac nghi¢m thoa man

2

diu ki¢n dang xet 111 nira kho.ing [ ; +00}

T6m li, tp nghim cua bat phuc.mg tr'inh (4) la

S= (-:; )u[ ; +oo) = (-:; +oo). O

**cau hol va bal ta**.**p**

1. Lp bang xet dau cua cac biu thuc :

a) 4 - *3x* ;

*2x* + l

c) *x(x* - 2)2(3 -x) ;

b) 1 - 2 - *x* ;

3x - 2

d) *x( x* - 3 2

)

(.r - 5)(1 - .r)

1. PhAn tfch cac da thuc sau thanh nhan tu bc nhat r6i xet dAu :

*2*

* 1. *-x*

*+ x + 6 ;* b) 2x2 - (2 + ./3).r + .Ji

1. Giai cac bat phuc.mg trlnh :

a) (3 - *x)( x* - 2) < 0 .

- '

*( x* + I

I

b) *5*

l - *x 2x* + 1

c) 12x - "'21+ hl2 -*x* I> *3x* - 2 ; d) I<*J2* - *J3)x* + • I s; *J3* + *J2..*

1. Giai cac h¢ bat phuc.mg trlnh :

(x - 3)(J2 - x) > 0

l*- x + 3 ;*

* 1. *4x* - 3

*-<*

2

b) {-2--\_-I $ 3 *x*

Ix I < t.

126

1



Scanned by Carritrca"tr'Zer &? **A**

,.... .



**Lovent8P**

' *I* &. % *· t* **:SZ C'UZC**

.l6. Gilli va bi¢n lun cac brt phttctng trlnh :

.,

a) *m..r* + 4 >' 2r + ,,,- :

2

b) *2mx* + 1 *x* + *4m* ;

2

c) .r(

')

m- - 1) <

4

*m* - I :

d) 2(m + 1*)x* s (m + 1)

*(x* - I).

.l7. Giii cac ba't phttong trlnh :

a) (-Jir + 2)(.r + 1)(4.r - 5) > O : b) 3 - *2x* < 0 ;

*(3x* - l )(x - 4)

-3.r + 1 ?

d) *x* + 2

s *x* - 2 .

C)

2.r + 1

<- -- .,

*3x* + 1 *2x* - 1

38. Giii va bi¢n lun cac ha't phttcmg trlnh :

a) (2r - fi.)(.r - m) > 0 : b) $ 0.

*J3 - x*

*x* - *2m* + 1

1. Ttm nghi¢m nguyen ci1a m6i h¢ bat phttong trlnh sau :

5 ' 1

6.r + - > 4.r + 7 15x - 2 > *2x* + -

7 b) 3

a)

-8.r + 3 2*x +* 25 :

-<

2

1. Giai cic phttcmg trlnh va ba't phtrcmg trlnh :

(.r \_ ) < *3x* - 14 .

2 .

2 4

b) 1 2.r - 1 1 1

a) l .r + l l + l x - 1 1 = 4 : (.r + l)(.r - 2) > 2·

1. Giai va bi¢n lun cac ti¢ bt phuO'Ilg trlnh :

2 5

f*(x* - J5)(J7 - 2.r) > 0

* 1. I

*l X - m O :* • ·

b) *x* - 1 < 2.r - I

{*x* - *m* 0.

127

Scanned by CamScanner



,.\_/ T TT *'fr-h...*TG TR'iNH vA

§ *5* BATPnuv1"I

, "CNl-IATHAIAi

G

.*·*;•

###### J-$BATPHl1C1N

TRINH B -

1. Bat phuang tr1nh bc nhat hai an
   1. Bat phuong tnnh bc nhat haa. a;n ,.a. mi.e.-n nghie.m cua no

DINH NGHlA

*BtiJ phuqng trlnh nhiil hai ti'n /a bill plut{fng trinh co mtj;*

*trong cc:ic dqng*

*ax* + *fry* + *c* < O. *ax* + *by·* + *c* > 0, *ax* + *by* + *c 5* 0. at *i by c* 0.

-,

*rrong do a. b ,.a c la nlumg so'cho truilc sao cho a- b- -z* 0 :

*x \·a y la cac an.*

*Mai cqp s6'Cx0* ; *y0) sao clzo ar0 by0 c* < 0 *g9 i la miJt nghi4m*

*cria bar plntcmg trinh* at + *by* + *c* < 0.

Nghi¢m cua cac ba't phucmg trinh dt:lng *ax .:.. by* + *c* > 0. *ax .:.. by c* :::;- G ,-a

at + *bj* + *c* 0 duqc djnh nghia tucmg ur

Nhu vy trong mt phang to d<), m6i nghi¢m cua bat phuong trlnh bc n a. hai an du<JC bieu dien ooi m(>t di m va l4P ngh i¢m cua n6 duqc bieu die bm m(?t tp hqp di m. Ta gQi p hqp diem ay la *mUn nghim* cua ;, phu<111g trlnh.

Oum day. chung ta se thay mien ngh i¢m cua bat phucrng trlnh b nhat hai an

1a m(lt mia mt phng.

1. Cach xac dnh mien nghim ciaa bat phuong tTinl) bc nhat haian

Vi¢c xac djnh mien nghi¢m cua m(>t bat phm:mg trinh bc nhat hai tin a

bieu dien hlnh hQC p nghi¢m cua n6) trong mi)t phing loa do dtta tren dinh h

. . . .

128

duqc thua nh sau day.

Scanned by Camscanner

*DINH Li*



-----...

*Trong mat /Jha:t11 d* , • *I* .

*h* • *dLt(Jng* + = 0 *c*

* + *g IO<J (), thcing (d) : ax + hy <.:· 11a*

*mavt p ano tha'i?I h* . •

·• *() 1 a1 nua m(vlt pha:n: g. M9*,*1 trong*

* • *v*

*/ za1 nua mgt*

*phang dy (khting kt/ hiJ (d)) gom cac die'm CO IO<J cl<) thod miin hat phum1g trinh ax + by + c* > O. *mra mqt phdng con lfli (/.:hong kl hiJ (d)) go'm cac dilm co IQ(/ cir) thod miin bat phU{.fflg trinh*

*ax + by* + *c* < O.

*Tit* djnh J f, ta suy ra

*Neu (xo* ; *Yo) la nu)t nghi¢m cua hat plut(Jng trinh ax + by + c* > 0 *(iiay ax* + *by* + *c* < OJ *thi mra m(it phdng (khong kl biJ ( d)) chua dilm M(x0* ; *y0) chinh la mie'n nuhi¢m nia bat pl 11rm1g trinh dy.*

V\*y d xac djnh mien nghi¢m cua *bat* phuang rrinh *ax* + *by* + *c* < 0, ta lam

nhtt sau :

- *ve-ditilng thdng (d)* :*ax + hy* + *c* = 0 ;

-*XCt nu)t dilm M(x0 :* Jo) *khong nam tren (d).*

*Neu a.x0* + *by0* + *c* < 0 *rhi m(a mq1 ph dnx ( khong kl biJ* (d)) *clura*

*diem M Iii mie'n 11ghi¢m ('[ ia bat pl 11t(J11g 1ri11h ax + by + c* < 0.

*Neu ax0* + *by0* + *c* > 0 *1hi mi'a m(it phdng (/.:hong kl hil* (d)) *kh6ng*

*chua dilm M /0 mie'n 11ghi¢m nia bat pl 11tong 1rinh ax* + *by* + *c* < O.

CHO Y

*D6i* v6i cac bat phuong trlnh dmg *a.r* + *by* + *c* 0 ho{lc *ax* + *by* + *c* O

thl mi6n nghi¢m la *mia* mt phng ke ca *ba.*

*Vi* d1:1 1.Xac djnh *mien nghi m* cua bat phttang trinh 3.r +*y* 0.

*Gidi.* Tren mt phAng to d¢, duang thang (d) : *3x* + *y* = O chia m t phc1ng thanh hai nfra mt phang.

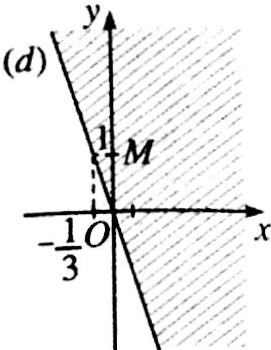
g. fliS6 10·NC·A

129

Scanned by Camscanner



Cbqn m¢t diem bat kl kh6ng thu ducrng thang d6, chang h diem *M(O* ; I). Ta thay (0 ; I) khong phai Ia nghiem cua bat phucmg trlnh da



cho. v y mien nghi.¢m can tlm fa nlta mt phAng

bO (d) khong cht!a diem *M(O* ; I ). (Tren hlnh 4.5,

mien nghi¢m la m1a mt phg kh6ng bj g<;tch). O

IH 11 *Xac dinh mien nghim cua b5t pht!Ong trlnh x* + *y* > 0. *Hinh 4.5*

2. H bat phuong trlnh bc nhat hai an

Oum day la m¢t vf d9 e *h¢ bat phuang trinh bt;ic nhat hai an*

*3x -y +3 > 0*

(I) *-2x +3y -6* < 0

*12x +y +4 >0.*

Trong mt phcing to d¢, ta gqi tp hqp cac diem c6 to de) thoa man m9i bat

phucmg trlnh trong h¢ la *mien nghim cua h.* Vy mien nghi¢m cua h la

giao cac mien nghi¢m cua cac bat phucmg trlnh trong h¢.

De xac d!flh mien nghi¢m CUa h¢, ta dung phucmg phap bieu dien htnh hQC

nhu sau : ·

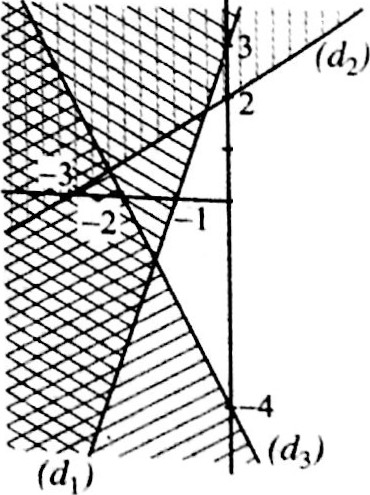
* *Viii moi bat ph U<lng trinh trong h¢, ta xac df nh mien nghief m c1.ia*

*n6 va ggch bo mien con l{li.*

- *Sau khi tam nhlt tren ldn IU{ft do/ viii tat cd cac bat phuang trinh trong h¢ tren cimg m<?t m{it phdng to{l d<), mien con f<;1i kh6ng bf ggch chinh ta mie'n nghi¢m cua h¢ bat ph U<Yng trinh do cho.*

Vi d1;1 2. Xac djnh mien nghi¢m cua h¢ bat phuong trlnh (I).

*Gidi.* Truac het, ta ve ba ducrng thng : *y*



*(d1 )* : *3x* - *y* + 3 = 0 ; •

(d2) :-2r + *3y* -6 = 0 ; (d3) : 2r + *y* + 4 = 0.

*0 x*

130

Thlr trt C tiep ta thay (0 ; 0) la nghi¢m CUa ca ba

bt phucmg trlnh. Dieu d6 c6 nghia ta g6c to d() thu¢c ca ba mien nghi¢m ciia ba bat phuong trlnh cua h¢ (I). Sau khi gch bo cac mien kh6ng

thich hqp, mien kh6ng b! g<;tch tren hlnh 4.6

(kh6ng k bien) la mien nghi¢m cua h¢ (I). D *Hinh 4.6*

9· OiSO 10 ·NC·B

Scanned by CamScanner

[ff2J *Xac djnh mien nghiem cua hA b ¥* •

* + 9' *at pht./Ong tnnh*

*J y - 3x* > O

*x - 2y + 5 < 0*

l

5x + 2y + l0 > 0.

1. M()t vi d1:1 ap d1;1ng vao bai toan kinh te

van de t m mien nghi¢m cua h¢ bat phtr<111g trlnh bc nhat c6 lien quan cht che den *Qu) hogch tuye iz tinh.* D6 la m(>t nganh toan hQC c6 nhic!u U'ng d9ng trong dbi song va kinh te. Sau day Ia m<)t vf d9 dan gian. ,

Bai toan

Ngum ta dt! d!nh dung hai IOC;li nguyen li u de chiet xuat ft n'hat 140 kg chat *A* va 9 kg chat *B.* Tu m6i tan nguyen li¢u IOC;li I gia 4 tri¢u dong, c6 the chiet xuat dugc 20 kg *chfit A* va 0,6 kg chat *B. Tit* m6i tan nguyen li¢u loi fl gia 3

triu d6ng, co the chjet xuat duqc I0 kg chat *A* va 1,5 kg chat *B.* H6i phai dung bao nhieu tan nguyen Ii¢u m6i Ioide chi phi mua nguyen Ii¢u Ia ft nhat, biet rg co so cung cap nguyen li¢u chi c6 the cung cap kh6ng qua I0 tan

nguyen li¢u Iocµ I va khong qua 9 tan nguyen li¢u loiII ?

*Phan rich bai loan.* Neu SU dl;lng *x* tan nguyen li¢u loi I va J tan nguyen li¢u loi II thltheo gia thie·t , c6 the chiet xuat duqc *(20x* + I*Oy)*kg chat *A* va (0,6.x +1,5y) kg chat *B.* Theo gia thiet. .r va *y* phai thoa man cac di6u ki¢n :

0 *x* I 0 va 0 *y* 9 :

|  |  |  |
| --- | --- | --- |
| *20x* + l *Oy* | 140, hay 2r + *y* | 14 ; |
| *0,6x* + *l .5y* | 9.hay 2r + *5y* | 30. |

T6ng *s6* ti n mua nguyen li¢u la T(.r :*y)* = *4x* + *3y.*

Bailoan da cho *trd* thanh :Tim cac so *x* va *y* thoa man h¢ bat phtr<111g trlnh

0 .r 10

*O y 9*

(II)

2.r + *)'* 14

*2x* + *5y* 30,

sao cho *T(x* :*y) = 4x* + *3y* c6 gia tr! nho nhat.

13 1

Scanned by camscanner

R\i tonn nlY dltn den hai bai toan nho sau :

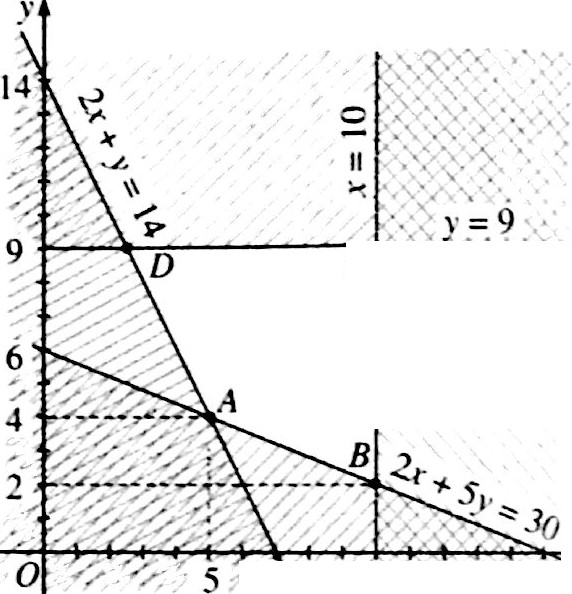
*B(li to,ln 1.* Xac nh t p hCJp (S} cac dim c6 to de) (.r ; *y)* thoa man h (II).

*Btli totln 2.* Trong t lt ca cac dim thu<)c (S). tlm diem (.r :\_v) sao cho T(.r :*y)*

(·6 gia trj nho nh!it.

* V ic giai ba.i toan l chfnh la vi¢c xac djnh mit!n nghim cua h¢ bat phtrong trlnh (II) ma ta da lp dtnJC.

IH3J *Kidm tra li rimg mien nghim*



*c*

' 7

10 ..> .

*1:*

\S) *cua hf!* (II) */a mien t(I giac ABCD tran hlnh 4.7 (keca bifJn).*

I

* giai bai toan 2. ta thtm nhn r g biu thll'c T(.r : y) c6 gia ttj

nho nhat va gia ttj ay d<;tt duqc tf:li

m(>t trong cac dinh cua tU giac

*ABCD* (xem bai dQC them trang

133). BAn.g...

each tlm toa.

do.

cac

dinh *A. B. C. D* roi so sanh cac gia ttj tucmg t.lng cl.ta T(.r : *y).* ta duqc *T(5* :4) = 32 Ia gia ttj nho nhat.

*x*

*Hinh 4 .7*

V Y de chi phi nguyen li u ft nhat, ciin sir dmg 5 tan nguyen li¢u Ioi I ,.a

4 tAn nguyen li¢u lo<;ti II (khi d6, chi phf t6ng c(mg Ia 32 trie.u dong....).

**cau hoi va baltip**

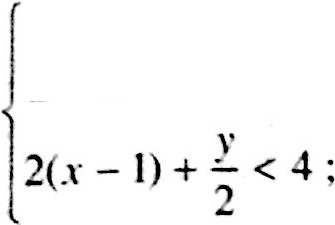
•

* 1. Xac djnh mien nghi¢m cua m6i bat phucmg trinh hai an :

a) .r - 2 + 2(v - I ) > 2r + 4 : b) 2t *-fi.y* + *Ji* -2 o.

* 1. Xac djnh mien nghi¢m cua m6i h bt phucmg trlnh hai an :

*x* \



- + -=- - 1 > 0

* 1. 2 3

4.r - *Sy +* 20 > O

b) \' > 0

-\' + 5 >

*x* - 3.

3

132

Scanned by CamScan ner

' ·-- ---·-·.... \_, -- -...

1. M('t gia dlnh can ft nhfi't 900 don vi prc'.'>t in va 400 d<Jn vj Jipit Lrong t h&tC an m6i ngay. Moi ki l6gam thit bo chi.la 800 dan vi pr6tin va 200 don vi lipi1. M6i kil6gam t hjt lqn (heo) chi.la 600 dan vj pr6tcin va 400 d<m vj l ipi t. Bie'I

rAng gia dlnh nay chi mua nhieu nha't la 1,6 kg thit bo va I,J kg th *it* Jqn ;gi

tien I kg thit bo la 45 nghln d6ng, 1 kg thit Jqn la 35 nghln d6ng. Gi' sll

gia dlnh d6 mua *x* kilOgam th jt bo va *y* kil6gam thit lqn.

* 1. Viet cac bat phttang trlnh biu th j ·cac dieu ki¢n cua bai toa thanh m()t h¢

ba't phttCIIlg trlnh roi xac djnh mien nghi¢m (S) cua h¢ d6.

b) G9i *T* (nghln dong) la *s6* tien !Jhai tra cho *x* kil6gam th it bO va *y* kilogam

thit lqn. Hay biu dien *T* theo x v *y.*

c) 0 cAu a), ta thay (S) la m(>t mien da giac. Biet rng *T* c6 gia tri nh6 nhat t<;ti .

*(xo ; Yo)* vm *(xo ; y0 )* la to d¢ cua m<)t trong cac dinh cua *(S).* Hoi gia dlnh d6 phai mua bao nhieu kilogam th it m6i loi d chi *phi* la ft nht ?



MOT PHUONG PHAP TlM *eve* TR! CUA s1Eu THUc

*P( ; y) = ax + hy* TREN MQT MIEN DA GIAC L61

BAI TOAN : *Tim gia tri nho nhfit hay Ion nhat cua bi6u tht1c P(x ; y) = ax + hy ( h* 7 *0)*

*tren m¢t mien da giac phAng J6i (kdca bienJ .*

Bai toan d6 c6 ngh a la :

. Cho bi u thlrc *P(x ; y) = ax + by (h* 0) va m<?t mien da giac 16i *(S),* ke ca bien, trong mt phang to d(> *Oxy.* Hay tim gia trj nho nhat (hay Ion nhat) cua *P(x* ; *y)* vai *(x* ; *YJ* la

toa. dO. cua cac diem thu¢c *(5).*

*Cach giai.* Ta luon c6 the gia thiet rng *b* >.0, bai vi neu *h* < 0 thita c6 the nhan ca hai ve voi -Iva bai toan tim gia trj nho nhat (hay Ion nhat) cua *P<x* ; *y)* se tra thanh

bai toan tim gia t Ion nhat (hay nho nhat) cua *-P<x ; Y> = -w:*+ *l/ y,* trong d6 *b'* = *-h* > o.

133

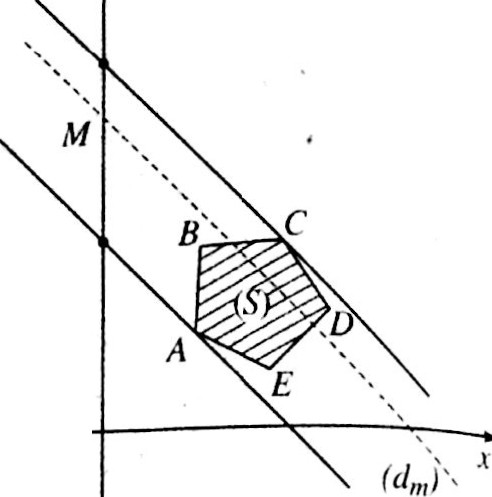
Scanned by Camscanner



Tp cac diem *tx* ; y) de *P<x* ; y) nhn gia tri *p* ta *)'*

d . , *a p*

l./Ong thang *ax* + *hr* = *p,* hay \' = --*x* + -. E>Lrang



*0*

. . *b b*

thng nay c6 he so g6c bmg *\_!!\_* va ct true tung

. *b* .

ti di m *M(O* ; 111) vai *m* = (h.4.8). Ki hi u dlle)ng

thng nay IJi (*dm* ). Vl *h* > O nen vi c tlm gia t nho nhat· (hay Ion nhat) cua *P(x* ; y) = *p* voi *(x* ; \_\') E (S) quy Ve Vi C tim gia tr! nhO nhat

(hay Ion nhat) cua *m* = *P tuc* la tim diem *M a*

*b '*

v! tri thap nhat (hc1y cao nhat) trim trl:'c tung sao cho dt.Jong thang (d"') c6it nhat m¢t diem chung voi(S).

*Hinh 4.8*

Tt d6. chu y rang *(d* ) c6 he so g6c bflng *\_!!\_* kh6ng d6i. Ta di den each lam sau :

*h*

*m* .

* Khi tlm gia tr! nho nhat cua *P(x* ; *y),* ta cho dt.Jang thang *(d,,, )* chuyen d¢ng song

song voi chinh n6 tlt mc)t v! *tri* nao d6 *a* phia dt.Joi mien (S) va di len cho den khi *(dm )*

Ian dau tien di qua m<?t diem (Xo ;Jo) nao .d6 cua (S). Khi d6, *m* d t gia tr! nho nhat

va tt.Jang (Ing voi n6 la gia t nho nhat cua *P(x ;* y).£)6 la



* Khi tim gia t Ion nhat cua *P(x* ; *y),* ta cho dt.JC1ng thang (d,,, ) voi ht} so g6c - chuyen dc)ng song song voi chinh n6 *W* m(>t vi tri nao d6 (J phia tren mien (SJ va di

xuong cho den khi (d"') Ian dau tien di qua m¢t diem (x0 ; *y0 )* nao d6 cua (5). Khi

d6, *111* d t gia tr! ton nhat va tt.Jong Lrng voi n6 la gia trj Ion nhat cua *P(.r* ;\_v). D6 la

P(.ro :Jo ) = *axo* + *hYo·*

CHU Y

Qua each lam tren, ta thay rang *P(:r: ;* y) d t gia t nho nhat (hay Ion nhat) tc;ii m(>t dinh nao d6 cua da giac *(S).*

134

I

Ap dl;Jng each lam tren vao bai toan 2 neu trong §5, ta thay khi *(d,,,)* di qua dinh

*A( 5* : 4) thi *m* nho nhat. Dieu d6 c6 ngh a la *T(x* ; *y )* d t gia tr! nho nhat khi *x* = 5 *va*

*v* = 4. Khid6, *T(5 ;* 4) = 32.

Scanned by camscanner

**Lurtn 18P**

4. X:ic tf j 11h mi n nghi'Ym dm r:k hll'r phuo11g 1rl11h hai 11 :

n) ·' + J I·2(2y I 5) 2( I ·') ; h) (f 1 *Ji )* . *( I Jj)y* 2.

*.a6.* Xtk djnh mi 11 ll!!hi 111 cuu c:lc h h[lt ph1ra11g 1rlnh hui rin :

.\ - v ,()

u) .r -3y j

*.r +y ,5 :*

*1x* 2 *v* · 6 0

*j*

b) 2(" I).. *2* \_,.4

2

*x* 0.

1. U9i (S) la l p h<,1p *c:k* rliCrn t roug m r phrlng 10 dO c6 10;.1 <1 1hoc'iman h¢

2.r *y* >- 2

*x - 2y* 2

*.r +y 5*

*x :?. 0.*

* 1. Hay xac djnh (S) d6 rhay rfmg d6 la m(H rni n ram giac.
  2. Trong *(S),* hay rim diem c6 10 d(> *(x ; y)* Jinn cho bieu 1huc*J(x : y)* =*y -x* c6 gia trj nho nhar. bi I n1ng./{.r ;*y)* c6 gia lrj nho nha't tt,ti rn¢t trong t:ac dinh cua (S).

1. *Bai 1oclt1 vitamin*

M¢t nha khoa hc,>c nghi n cU'u v tac d<)ng phOi hqp cua vitamin *A* va vitamin *B*

d6i v6'i*ca* the con nguoi. Ke't qua nhu sau :

1. *M()t* nguO'ic6 th6 1iep nh n duqc mc\ingay khong qua 600 don vj vitamin *A*

va kh(}ng qui 500 dl111 v j vitamin */J.*

1. M<)t nguO'i rn6i ngay cdn tl! 400 d6n I000 don vj vitamin ca *A* hln *8.*

iii) Do rac d(lng phoi *hqp* cua hai loi viramin, rnbi ngay, s6 dan v! vitatnin *B*

khOng ft hem s6 don vi vitamin *A* nfumg khOng nhi u hem ba IAn s6 don v!

vitamin *A.*

Gia su*x* va *y* Ian IU'qt la *s6* dcm vj vitamin *A* va *B* ma bn dong m6ingay.

a) Gqi*c* (d6ng) la *s6* rien vitamin m.a..bn!)hai.tn1 m6ingay\_. Hay viet phucrng trlnh bieu di n ,. dtr6'i dmg m<)t b1tu thtrc cua *x* va *y .* neu gia ml>t ( O'n vi vitamin *A* la 9 d6ng va gia m(>t dcm vi vitamin *B* la 7,5 d6ng. . .

135

Scanned by camscanner

h ) V1 •r *c:k* h. I pf nmn • Ir n h bi ,u r

. *r.*c

J' ·u k i·n i) *i i )* · vii iii) rha11h fl1r,1 Ii ·

c h! c.a t 1 • . ,

h1

hfi'1 phu11n , lf i n h roi x6c <l!n h n11.'·n 11g

. ·n1 *( ·*c*'*·*>*

. • . he t,rrr phwmg r rmh d<'·

<.: u. · •· .

1

c l ·nm J'lnro11g 1111 dun

. . v.

rn . " . // r ho:i rt1•i11 ttk di u k1 ,'n t r6n clc

ha1 loa1

11a

in " v.1 •

" 1il- 11 pli:ii t rri 1:1 11 ""·I.bic·r ;mg < <Ii.t i gi{t r r! nh(,nh.ir ll,J i ml)t r rong *cr.c* dinh

cu:1 1111 -: 11 nghi rtl *(S) .*

**VAi NET v LICH sll QUY HOCH TUYN TfNH**



'lf tho• v do1. khl t hllc h1en cac c6ng *vi c* cua minh. 1001 n *1.J* , c1a lu6n hJong tor each lam *t6t* nhat trong c/Jc f1ct1 lttm o6 th cJllOC (t1m phl/Ong an t6i lJU trong



c!1c ptn1on an) Kh1 toan h9C phat tri n. nguoi ta da rnO h1nh hoiJ to:Jn hoc cac v1ec can lam. ngh a la bit3u lt11 <- m JC t1 U n dat dl/Qc, cac yeu cau hay CaC d• ·u kr .:n c!.n tt 10 m:.n b ng ngon ngii toan h<;>c d tirn '"' *gil11* t61 LIU cho n6 *Tt.1* d6. hinh thanh nen cac bal to{,n t61 11u

*Ouy hoai h tuy n tmh* la fmh *vt.Jc* toan hQc nghien c(tu fie t>b 1 roan t61 lJ VOi hl/u hn bren (an). trong <16.

rrwc 11 u v&J c.,fJc di u kr n rang bu(>c du c biliu th! *Kup-man, Dan-dich va*



btfn' cfJc h"•m *'.6.* ca phllcmg trinh hay bat phllOng *Kan-to-r6-vich o*

1ru1h tuy n tinh (bfic n *) Luc-xtJm-bua ntJm 1976.*

C6 th n61. ngllcJ1 "au ti "'n quan lc1m d6n *Ouy ho9ch tuyen tlnh* la L. v. Kan-to-r6-v1ch

.

(L1'°n1d V1tnlyev1ch Kantorov1ch, 1912 1986). Trong *cu6n* "Cac phlldng phap toan

.'

• ' ho , tron *t6* ch(lc *Vi)* k hOt=JCh hoa san xu#ft" (NXB DC;Ji h<;>c *Ou6c* gia Le-nin-grat.

1'139), Ong c1a nu bat Vol tro cua mot *lbp* bai toan *Quy ho9ch tuyen tlnh* va de xuat

f

' th *1 t* totln *o* b¢ d g1a1 chung. Tuy nhi n. Ouy hoC;Jch tuyen tfnh chi du9c nhreu

ngllf.11 b1'1t *d 6n 1ao* nam 1947 , kh1 G.8. Dan-dich (George Bernard Dantzig, 1914

2005) *cX>ng b6* thuf.tt toan don hinh d grai cac bailoan *Quy ho{Jch tuyen t/nh.* Cung rH rn du, i C Kup-man (T1alling Charles Koopmans, 191O - 1985) da chira r ng

*Ouy flUt)Ch tuydn tlnh* la *wng* Cl) tuy t vcJi " ph:\n tich Iithuyet kinh te co di n.

*'4£Jm* 1975. Kan tg.r6-v1ch va Kup-man da llQC Vi n Han lam Hoang gia ThuY. Oi n

*f !80* gt!tj thl.lt'Jnq *r* '0-bf;:n ve khoa h9C kinh te.

,,,; *:1y* nay, trc,nr the11 tJai may tlnh d1f;n h . *Ouy hoach tuyen tlnh* van *<1llOC* tiep tuc

*ngh,n (,(/JJ* nhrtrn tlrn ra c,ac thuat toan *t6t* htln . ·

Scanned by CamScanner

**•W &ul"'im - *a*** -.; ..

b) Vi t cac b£1t phltO'ng trlnh bieu t h! cac h.;U tyn 1 • ' , ,· .

, . . , d ·;. k .A ·) ii) v • iii) t hanh m01 he

bt. phuang t rlnh r6i xac d!nh

\t

. . .

mien ngh1

(" .• . he bff ptnrcmg t n nh do.

.1) cua . ··

m

c) nm phuang an dung hai IO<\_ ·v· · *A*

,*B* th *:* nan cac di u kicn t ren.de

ll itamin va

o..t I . . ,

so- t1·e·n pha,1. t ra•

la, 1,t nha,t , bt.t .t ra·ng *c* d<.•t g·t c*.1*t tr·. nho' nhat ta·t mo·t trong cac d1nh

cua mi n ngh i¢m (S).



**VAi NET VE L**.**ICH Slr QUY HOA**.**CH TUYEN TiNH**

Tl/ thdi co di. khi thlfc hi n cac cong vi c cua minh, loai ngLtoi da luon hLtong tai each lam t6t nhat trong cac each lam co the dLtc;1c (tim phLtong an t6i LtU trong cac phLtang an). Khi toan h<;>c phat trien, nglfdi ta da mo hinh hoa toan hoc cac viec can lam, ngh a la bieu thj cac myc tieu ca d t dLtQC, cac yeu cau hay cac dieu ki n can thoa man bng ngon nglJ toan h<;>c de tim ldi giai t6i LtU cha n6. Tl! d6, hinh thanh nen cac bai toan t6i LtU.



*Quy hoch tuyiin tinh* la finh vlfc toan h<;>c nghien cl/u cac bai toan t6i llU voi hl!u hn bien (n), trong d6,

myc tieu va cac dieu ki n rang bu<;)c dLt<;jc bieu thj *Kup-man, Dan-dich va*

bang cac ham s6, cac phLtong trinh hay bat phLtong *Kan-to-r6-vich* d

trin tuyen tlnh (bc nhat). *Luc-xtJm-bua ntJm 1976.* C6 the n6i, nglloi dau tien quan tam den *Quy hoch tuyen tlnh* ia L. v.Kan-to-r6-vich (Leonid Vitalyevich Kantorovich, 1912 - 1986). Trong cu6n "Cac phLtong phap toan h<;>c trong to ch(/c va ke hoch hoa san xuat" (NXB E>i hQC Qu6c gia Le-nin-grat. 1939), ong da neu bt vai tro cua m<;)t lop bai toan *Quy hoch tuyen tinh* va de xuat

thut toan so b<? de giai chung. Tuy nhien, Quy hoch tuyen tinh chidLt9c nhieu

nglfdi biet den vao nam 1947, khi G.B. E>an-dich (George Bernard Dantzig, 1914 - 2005) c6ng *b6* thut toan don hinh de giai cac bai toan *Quy hoch tuyen tlnh.* Cung nam d6, T. C. Kup-man (Tjalling Charles Koopmans, 1910 - 1985) da chira rang

*Quy hoch tuyen tfnh* la cong cy tuyt voi de phan tich Iithuyet kinh te co dien.

Nam 1975, Kan-to-r6-vich va Kup-man da dLtc;1c Vi n Han lam Hoang gia ThuY. £)ien trao giai thLfOng No-ben ve khoa hQC kinh te.

Ngay nay, trong thoi d i may tinh di n *hi, Quy ho[!ch tuye'n tlnh* van dLtOc tiep tl}C

nghien cl/u nhm tim ra cac thui[lt toan t6t hon. ·

136

Scanned by CamScanner

DAU CUA TAM THVC BAC HAI



H' ? ,

1. Tam thuc bc hai

DJNH NGHIA

*2*

*Tam thitc bc hai (dol rl!i x) la. bilu 1/11fc dqng llx*

I*trong do a, b, c la nhimg st/ cho rn1ifc 1·1!i a -:t:* 0.

+ *bx* + *c,*

Theo djnh nghia tren, cac bi u thuc

fi..r) = *-fi.x 2* + 3.r + I , *g( x )* = *x 2* - 5

la nhiing tam thuc bc hai.

va *h(x) =*

I ..,

*-x-*

2

Nghi¢m cua phuang trinh bc hai *ax2* + *bx* + *c* = 0 ding duqc g<;>i la *nghi¢n1*

*2*

*cua tam thuc b(u: hai j( x)* = *ax*

*+ bx + c.*

cac bieu thUC */1* = *b2* - *4ac* va *11'* = *b'2* - *QC* vffi *b* = *2b'* theo thU fl! ciing dl!gc

*2*

g<;>i ta *bi¢t th1tc n1bi¢t rhfrc rlw g9n di.a ram thuc b(lc hai* f(.r) = *ax*

*+ bx + c.*

Trong §4, ta eta xet ctau cua nh!thuc b c nhat va ap dl;lng d giaj m¢t s6 bat phtrong trlnh. Trong bai nay \·a cac bai tiep theo cua chucmg, ta se xet dau cua tam thuc bc *hai* va ap d1.:1ng n6 de giai cac bat phuang trinh va ph ucmg trinh b hai ciing nhu m<?t *s6* phuang trlnh va bat phucrng trlnh khac.

1. Daucua tam thuc bc hai

Ta se quan sat do th! cua ham *s6* bc hai de suy ra d!nh If v dau cua tam thuc

bA.c ha1" /(.\.) = *ax*2 + *b*.\*.* + c.

Du cuaf(x) ph thu()c vao da'u cua bi¢t thuc */1* va h¢ *s6 a.*

137

Scad15y camscanner

'

- . .

Trong tltllg tnrang h<Jp, dAu ,

) d qc n

*A* tfOOg caC bang SaU :

u

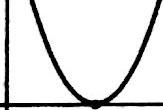
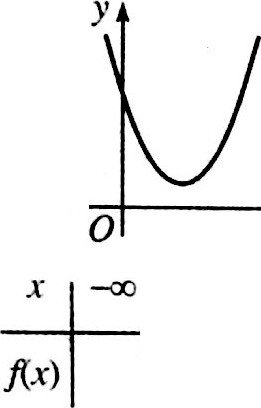
cuaftx u

I ) *L\* < 0 (tam thuc bc hai vo nghi¢m)



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *a> O* | | *a< O* | | *Ket lu(m* |  |
| + | *x*  - • | *y 0*  *x* 1-00  *ft.x)* | *x* | */(.x)* Cung da'u vcri *a*  *(aj(x)* > 0 vai mqi*x* e IR). |

2) *ti.* = 0 (tam thUc bc hai c6 nghim kep *xo* = -*;a)*



*a> O*

*a< O*

*Ket lu(m*

*y y*

*X'*

*x*

*ft.x)*

-00

*Xo*

+oo

*0*

*x* 1-00

*ft.x)* +

*Xo*

*Xo*

*x*

CUng dau

vm *a*

0

I vm *a*

-ngdau

+oo

*Xo*

+oo

0

+

*x* 1 -00

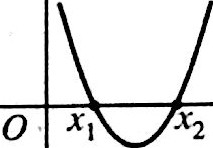
*j(x)*

*aft.x)* > 0 v(1i *mqi X :t Xo).*

0



*y*

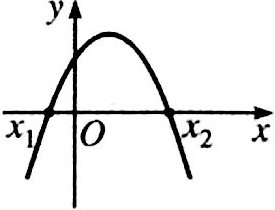


*a> O*

*a< O*

*Ket lut;ln*

j(x)



*x x* -oo *x 1* x

Cung dtiu I Khac dAuICung

*ft.x)* vm *a* 0 vm *a* 0dffu vai *c*

I I i

--+- --

2 +oo *(aftx)-<* 0 vm ffiQi *X* E(.r1 ;*xz).aj{x) ;;.a*

+ 0 - 0 +

*fix)* - 0 + 0 - v6i mqi *x* E(-«> ;x1) u (x2 :+oo)).

138

**:r-t r;;;-.--•ane:rJ!lFi'.''" - ...----r** ,.**.a ....-.--..** . ·

Scanned by CamScanner

Qic ket qua tren duo.c phat biu trong d'nh 11, sau day.

D!NH Li (ve dau cua tam thllc bc hai)

*Cho tam tl1*14*·*1*c·b*

*{le*. *ha1*'f(x) = *ax*2 + *bx* + *c (a -:t* 0).

*N eu 6 < 0 thifi"*\-"*'·) c·u'ng da·'u vm..h'! s6*-*a vC*.*//*.*mr;n*.*x* E TD> .

)A

*Nlu* 6 = 0 *thif( x) cung da'i1 viii he so'a viii moi x -:t*

*\_ }!\_ \_*

· · 2a

*N eu* 6 > 0 *thi* f(x) *c6 hai nghi¢m X i va* X2 *(X 1* < *X2 ). Khi d6 , fix ) trai ddt1 viii h¢ s6' a viJi m9i x nam trong khodng (X* 1 ; X2) *( IUC fa viii* XI < *X* < *X2 ), VG* f(x) *Cung ddu viii h¢ so' a viii m9i X nam ngoai do<;1n* [x1 ;x2] *(titc la viii x* < x1 *hoc x* > *x2).*

CHO Y

Ciing nhu khi giai phuang trlnh bc hai, khi xet dau tam thuc bc hai, ta c6 th dung bit thuc thu g<;m *s* thay cho 6 va ding duqc cac

ket qua tuang *tl!·*

Vi **dl,l 1.** /(x) = 2\ 2

-

va *a =* 2 > 0.

-*x* + 1 > 0 vai m9i*x* E JR vi tam thuc /(x) c6 6 = - 7 < 0

0

Vi **dl,l** 2. Xet dau cua tam thuc bc hai *f* (x) = *3x2* - 8x + 2.

4 'IO 4 + 'iO

*Gidi.* Vi *a* = 3 > 0 va *j(x )* c6 hai nghi m *Xi* = -*:*l U ' *X2* = 3"l U

(de thay *X i < xi)* nen j{.x) > 0 (cung dau vffi a) khi X E (-oo ; X1) U (X2 ; +«>), Va

f(x) < Q (trai dau VOi *a)* khi *X* E (X1 : *X 2).* 0

Cung c6 th ghi ket qua tren trong bang xet du cuaf(.r) nhu sau :

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| .r | -00 | *x* I |  | *X2* | '  +oo |
| *j( x)* = *3x2* - 8x + 2 | + | 0 | - | 0 | + |

**[H** *Xet dfiu cua cac tam thtic bc hai sau :*

*a)* /(x) = -2x2 + *5x* + 7 ; *b) g( x)* = -2x2 + *xf5* -7 ; *c)* h(.r ) = *9x*

*2*

- 12..r 4.

139

Scanned by Camscanner

***Nhn*** *xet*

*Tit* dinh Iive du cua tam thuc bc hai, ta thAy chi co m(>t tnremg hqp duy nMt trong do da'u cua tam thuc khong thay d6i (lu6n Am hoc lu6n dtrong), d6 Ja khi *L\* < 0. Luc do, dA'u cua tam thuc trung vm dau cua h¢ *s6 a.*Do do, ta c6

2 *{a >* 0

V*x* e IR, *ax* + *bx* + *c* > 0 *L\* < 0

*Vx* E IR, *a.x*2 + *bx + c* < 0

*{a <* 0

*L\* < 0.

***i* dl;l** 3. Vm nhii'ng gia tr!nao cua *m* thl da thuc *ft.x)* = (2 - *m).x2* - 2r + I lu6n dtrong ?

*Gidi.* Vm *m* = 2 thl *ft.x)* = -2x + la'y ca nhii'ng gia tr! am (chang hcµt

*f* (I) = -1). Do do, gia tr!*m* = 2 khOng thoa man dieu kin doihoi.

Voi *m -:t:-* 2, *f( x)* la tam thuc bc hai voi bit thuc thu gQn fl' = *m* - I . Do d6

*Vx,fix )* > 0 e:>

*{a* = 2 - *m* > 0

e:>

< 2 -

*{m*

e:> *m* < I.

·= m - 1 < 0 m < l

Vy vm *m* < 1 thl da thucf(x) luOn dtrcmg. O

IH2l *Voi nhrmg gia trj nao cua m, da thtic f ( x)* = *( m* - l)x2 +*(2m* + l)x + *m* + I *am voi*

*m9i x thuCc* JR *?*

**cau hoi va bat tap**

•

* 1. Xet da'u cac tam thuc bc hai sau :

a) *3x2* - *2x* + I;

c) *x2* - *..f3x* + ;

4

b) *-x*2 + *4x -* l ;

d) (l-./i.)x2 - *2x +* I+ fi..

**SO.** Tim cac gia tr!cua *m* de m6i biu thuc sau luOn dtrong :

.x

* + 1. ( 2

m

+ 2) 2

- 2(m + *I)x* + 1 ; b) *(m* + 2)..r2 + *2( m* + *2)x* + *m* + 3.

140

Scanned by Camscanner



Cl. 'luu \,' k i:iIIitarn *111* l1e' m0i b'*:\* .

* ll".U Ihm:sau luon 3m :

•

) \ .. I *'111* 'l \' 1 *:.*

•' - - *..111* - I :

b) Vn -

..,

h- - 2(m - 3).r + m - I.

cz. Chtl'H minh ,f jnh If " lf \u \'th • L , •

1

* • ·"" liltrc br ha1.

*l/ llt.'11,i.: dt/n.* \ cl'i k lt1Jt'l'llµ h\lp < 0 \' *=*0.Slr dung he thUc da bift

l: .

"'-' .

/( \ *) ti* l(·r I :;*I*-'-J

- --\-,.*]*

hay aj(.r) *=* ,,.[ .r +

*b* J-..,

- --,J.

\_,, a- *1a 4a-*

Tnl\ltl h,.,.,.\ 0. sfr dmc h 1turc <l•1bi ·1

/tr> =- ,1{.r -.\ 1 )(.r -.r) hay ljl,r) = *a(.r* - .r1)(.r - .r::!).

t f'(lf1g tM .r, Vtt .r ltt ha i nuhi m cua lam link bc haij{x).

 BAT PHVO'NG TRINH BAC HAI

I. f>fnh nghiu \'1) t't('h gkai

*Btit fJl111011g tri11/ 1 h<jc llai (,iii .r ) lei hlif pl11t 0'11 tn'nh c6 m )r 1*n *mg cci(* ·*d lllg* .fl.r) > 0.j(..r) < 0../(.r) 0./(.r) - 0. *rrong d6*.fi..r) /d *m(il ltllll !lute / J<.h*·*lwi .*

*Cll.(' I*

*I*

*gl*. .. D . gt.a.a. t1•'\l f lhtr\11H!trlnh hi·l' hai.la •p du·ng.\_

di·nh If v d3u cua tarn

thm:hL) ' hui.

*( ll .* l

Vi d1:1 I.Gi•ii b!U ptunmg t rlnh

\_.r 2 - 1r I " 0. ( J

I I

Scanned by camscanner

1 '

*Gidi.* Tam thuc bc hai 2x2 - *3x* + *1* c6 hai nghi¢m *X i* = 2 va *x2* == I va co

h *s6 a* = 2 > 0 nn

2x2 - *3x* + I> 0 *x* < ho(ic *x* > I.

Vy p nghim ciia (I) Iii (-oo ; ) u (I ; + ro).

Ta biu din tp nghim cua (1) tren lfl;IC *"s6* (h.4.9). 0

..



I I

2

*Hinh 4.9*

IH1I *Tim t(lp nghim ciia cac bat pht.Jr:Jng trinh sau :*

*a")*

*x*"*·*? + *5x* +4 < 0 ; *b) -3x2 +2J3x* < I ;

*c) 4x -5* 7

3

2.

*-x*

1. Bat phuang trlnh tich va bat phuang trlnh chua n *iJ* mu thuc

Vi d1;1 2.Giai bat phuang trinh

*2x*2 *+3.x -2* > O

*x2 -5x +6* - .

*Gidi.* Ta xet dau cua bieu tht!c

*ft.x)* = *2x + 3x -2 .*

2

x2 *-5x +6*

*Ti'r* thtl'c la tam thuc bc hai c6 hai nghi¢m -2 va \_!\_ .

. 2

MAu thuc Ia tam thuc bc hai c6 hai nghi¢m 2 va 3.

Dau cua*f( x) .*duqc cho trong bang sau



|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | -2 | | | 1  -  2 | | 2 | 3 | +o'J |  |
| *2x*2+ *3x -2* | + | 0 | | 0 + | | + | | + |
| *x*2 - *5x* + 6 | + | | + | | + | 0 | 0 + |  |
| *ft.x)* | + | 0 | | 0 + | |  | + | |

142

Scanned by camscanner

*:* "

rap nghi¢m cua bat phuong trinh d- h '

* + a c o la

( - ; - 2] u [\_2!\_ '. 2)

u(3 ; +oo). 0

·***fB*** *Giai bat phl!Ong trinh*

(4 -2x)(x2 + 7x + 12) < O.

**Vi d1;1 3.** Giai bat phuong trinh

*2x2 -16x+* 27

2 2.

*x* -7x + IO

*Gidi.* Mt phucmg trlnh da cho tucmg ducmg v6i

*2x2* -16x + 27

2 - 2 0.

*x -7x+ I O*

(1)

Ta c6

*2*

. *2x*

*2*

-16.x+ 27 -2(*x*

-*7x +* IO) < -2x + 7 ::; O.

0) <=> 2 -0 - 2

*x* -7x+ IO *x -7x+ l 0*

.. *-2x* + 7 ·

=

Dau cua.f(x) duqc cho trong bang sau day.

*x2* - *7x* + IO

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* |  |  |  | |  |  | 7 |  |  | |
| -00 |  | 2 | |  |  | -  2 |  | 5 | |
| -2x + 7 | + | | | + | | | 0 | | | |
| *x2 - 7x* + 10 | + | | 0 | |  | - | I |  | 0 | + |
| *f(x)* | + | | |  |  | - | 0 | + | | |

Tp nghim cua bat phucmg tr1nh da cho Ja



(2 ; ]u (5 ; +«>). 0

3-. , **H bat phucmg trinh bc hai Vi du 4.** Giai h¢ bat phucmg trlnh

. *{3x2* - 7*x* + 2 > 0

(I) *-2x2* + *x* + 3 > 0.

143

Scanned by Camscanner

*Cch gidi.* Mu6n giai h¢ bAt phue111g trlnh bc hai m(>t An, ta giai rieng tung

bat phue111g trinh cua h¢ roi IAy giao cua cac tp nghi¢m tlm duqc.

Ba't phuaiig trlnh thU nha't c6 tp nghi¢m S1 = (--0p ; ) u(2 ; +oo).

Ba't phuang trlnh thU hai c6 tp nghi¢m S2 = (-I ; )·

Mu6n tim *s*I n S2 , ta c6 the biu dien cac tp nay tren tn.1c *s6* bng each Ian lugt g':lch bo cac phan khOng thu(>c S1 va cac phan khOng thu(>c S2. PhAn con li khong bi gch la *S* = *S*1 nS2 (h.4. 10).

I 3



-1

3 2

*Hinh 4.10*

.Ti'r d6 suy ra tp nghi¢m ciia h Ia S= (-1; }

Trong thtJc hanh, bai giai vf dv 4 thuang duqc trlnh bay nhu sau.

*Gidi.* Ta c6

*x <* Ihoac *x > 2*

(I) ¢::>

*-*3 .

3

-1*< x <-*

2

I

-l< x < -·

3

Tp nghi¢m cUa ba't phuang trlnh dii cho Ia khoilng (-I ; ). 0

IH3I *Giai h b5t phl.IOng trinh*

2x + l > 5

{2x2 -9*x* + 7 =:; 0.

Vi d1;1 5.TI cac gia tti cua *m* de bat phue111g trinh sa vo nghi¢m

*(m* - 2)x2 + 2(m + l)x + *2m* > 0.

2

*Gidi. Dtflx )* = *(m* - 2)x

+ 2(m + *l)x* + *2m.*

Bat phue111g trlnh da cho vO nghim khi va chi khi *f( x)* 0 voi mc;>i *x* E JR.

144

Scanned by CamScanner

V6i *m* = 2,ta c6./{x) = fu- +4.Khid6,Jlx)!> O x!> - .

Gia tti *m* = 2 khOng thoa man di u ki¢n doihOi.

v • *m*

2' ta co,*ft.x*) $ 0 v6i moi *x* E R khi va chi khi

{*!l* I 0,

6i

· *m - 2 < Q*

Thay fl' = *(m* + 1)2 - *2m(m* - 2) = -m2 + *6m* + I vao h¢ trtn,ta c6

2

-m + *6m* + I$ 0 <.:::> *{m* $ 3 - *JJO* hoc *m* 3 + *JJo* ¢::> *m* $ 3 - *J)o.*

{*m - 2 < 0 m < 2* ·

vy bAt phucmg trlnh da cho v6 nghi¢m 1khi va chi khi

*m* $ 3 - *JJO.* D

**cau hol vi bal ta**.**p**

1. Giru cac ba't phuong trlnh sau :

a) *-5x2* + *4x* + 12 < 0 ;

c) *3x2* - *4x* + 4 0 ;

1. Giru cac bat phuong trlnh sau :

b) l*6x2* + *40x* + 25 < 0 ;

d) *.i*-*x* - 6 $ 0.

"2 - *9x* + 14 0 *-2x2* + 7*x* + 7 < 1 .

* 1. ' ,

*x- ·- 5x* + 4

> ; b)

*x2* - *3x* - 10

4 2 0

- - '

c) (2t + l )(x2 + *x* - 30) 0 ; d) *x* - *3x* $ .

SS. Timcac gia tr!cua *m* de m6i phuong trlnh sau co nghi m :

a) *(m* - 5)x2 - *4mx* + *m* - 2 = O ;

b) *(m* + I)x2 + 2(m -1)x + *2m* - 3 = 0.

56. Giai cac h¢ bAt phucmg trlnh :

2x2 + 9*x* + 7 > 0

a)

{x2 + *x* - 6. < 0 ;

4x2 - *5x* - 6 $ 0

b)

{-4x2 + 12.x - 5 < 0 ;

2x2 + *x* - 6 > 0

*x-*

-2x2 - *5x* + 4 $ O

c)

{-*x2* - *3x* + 10 0 ;

d) {

*3* " - 1*Ox* + 3 > 0.

145

1. iS4 10·NC·A

Scanned by CamScanner

**Luven tP**

* 1. Tim cac gia ttj cua *m* d phtrcmg trinh sau c6 nghi¢m

*.x 2* + *(m* - 2).r - *2m* + 3 = 0.

* 1. Cluing minh rAng cac phtrcmg trlnh sau v6 nghi¢m du *m* lay bat kl gia tr!nao :

x

a) *x2* - *2(m* + I *)x* + *2m2* + *m* + 3 = O ; b) *( 2*

*m*

+ l ) 2

+ *2(m* + 2)x + 6 = 0.

* 1. lim cac gia ttj cua *m* de bat phtrqng trinh

*(m* - I)./ - *2(m* + l*)x* + *3(m* - 2) > 0

nghi¢m dung vm m9i *x* E IR.

* 1. Giai cac bAt phtrang trlnh sau :

*x4 - x2*

a) $ 0 ;

*x2* + *5x* + 6

b) ' I < I

*x-* - *5x* + 4 *x2* - 7*x* + 10

* 1. lim p xac djnh cua m6i ham *s6* sau :

a) *y* = *J( 2x* + 5)(1 - 2.x) ;

* 1. Giai cac h bAt phtrcmg trlnh :

*4x* -3 < *3x* + 4

* 1. *y* =

2

*x 2*

+ *5x* + 4

*2x2* + *3x* + l

* + 1. *{*
    2. {2.x

+ *9x* - 7 > 0

c) *{*

*x 2* - 7*x* + l 0 $ 0 ;

*x 2* - 9 < 0

*(x* - 1)(3.x2 + 7*x* + 4) 0.

..,

*x-* + *x* - 6 < 0 ;

1. lim cac gia ttj cua *a* sao cho V<Ji ffiQi *X,* ta lu6n c6

*x 2 +5x+ a*

-I :s; < 7.

*2x2 -3x+ 2*

1. Tim cac gia ttj cua *m* d h¢ b4t phucmg trlnh sau c6 nghi¢m :

*x 2* +*2.x* -15 < 0

*{(m+* l)x 3.

146

Scanned by Camscanner

1. Di*S6* 10·NC·B





*M9T* s6 PHUONG TRiNH VA . .



BAT PHUONG TRiNH QUY V BACHAI

< •

* 1. Phuong trinh va bat phucmg trinh chua an trong diiu ghi ttj tuy t *dm*

Vi d 1.Giai bat phuang trlnh *x1-* -*x* + I*3x* - 2 f > o.

*Gidi.* Truac het, ta bO dau gia ttj tuy¢t d6i.

Neu*3x* -22: 0 thl *x2* -*x* + I*3x* - 2 1 *= 1*-*x* +*(3x* -2) *=x2* +2r -2 ;

Neu *3x* -2 < 0 thi*x2* -*x*·+ j *3x* -2 !=*x2* -*x* - *(3x* -2)=*1-*-*.;x* -r 2.

-

Do d6,bat phuong trlnh da cho tuang duang v6i :

*3x* - 2 2: 0 *{3x* - 2 < 0

(I)

*{x2* + *2x* -2 > 0

ho¥: (II)

.,

*x-* -4*x* + 2 > 0.

N6i each khac, tp nghi¢m cua bat phucmg trlnh da cho fa hqp cic t?tJ nghl

cua hai h¢ bat phuong trinh (I) va (m.

Ta co

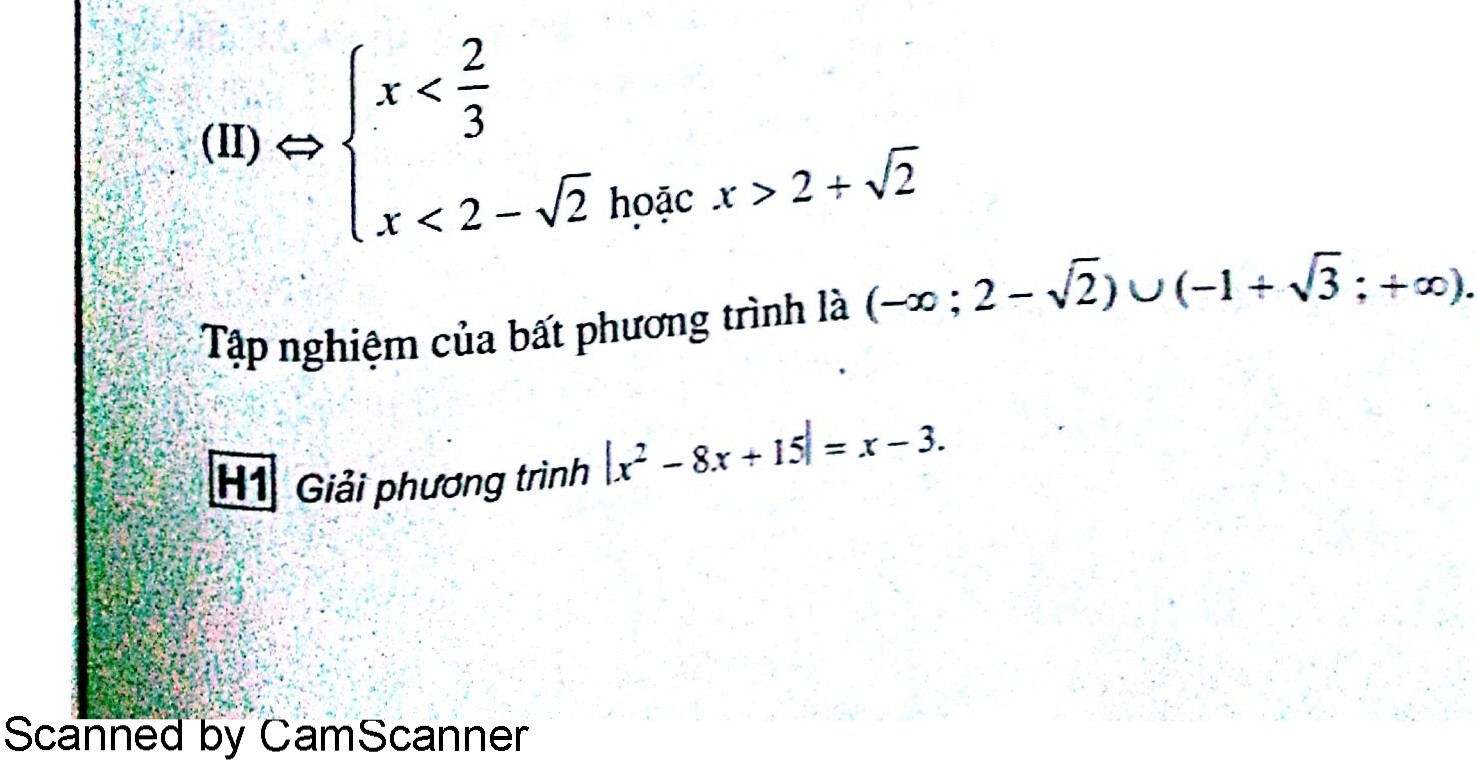
. '·

.... .

(I) ¢::>

2

*x > -*



3

*x* <-1 \_ Jj hoc *x* > -1 *J3*

<:::> x > -1 + -Ji

*x < 2 - fi..*

D

147

--

* 1. Phuong trinh va bat pbtrong trinb chua an trong dau can bc hai Khi giai phuong trinh hoc t phuong trinh chua an trong \_dau can bc hai, ta thl!c hi¢n m<)t *s6* phep bien d6i tucmg ducmg d dtta n6 ve m()t huong trlnh hoc bat phuong trlnh khOng con chua an trong dau can bc hat. Trong qua

trlnh bien doi cAn hru y :

* + - Neu cac dieu ki¢n xac djnh cua phuang trlnh hoc bat phuong trinh va neu dieu ki¢n cua nghim (neu c6) ;
    - Cfli binh phuong hai ve cua phuang trlnh hoc bat phuong trlnh

khi ca hai ve du khOng Am.

G¢p cac diu ki¢n d6 vcri phuong trlnh hoc bat phucmg trlnh mm

nhfµl dtt<JC, ta c6 m¢t h¢ phtt<111g trlnh hoc bat phucmg trinh tuong duong vm phuong trinh hoc bat phttong trinh da cho (rue la

ph ttong trlnh hoc bat phttong trinh da cho va h¢ thu duqc c6 cung

tp nghi¢m).

0 day' ta chi xet m<)t *s6* vf d1;1 don gian.

Vi d1;1 2.Giai phu<111g trinh *3x2* + *24x* + 22 = *2x* + I.

*Phan tich.* Diu ki¢n xac djnh cua phttong trinh da cho Ia

*2*

*3x* + *24x* + 22 O.

™thay nghi¢m cua phttong trinh da cho phai thoa man dieu ki¢n

*2x* + I 0.

Vm cac diu ki¢n ( I ) va (2), phuong trlnh

(I )

(2)

phtt<111g trinh da cho tuong duong v6i

*I*

2

*3x* + *24x* + 22 = (2x + 1)2. ,.,

II·e · (J)

t *nhin* (3) keo theo (I ). Do d6, nghiem cua , \_ . ngh1¢m cua phu<111g trinh (3) thoa man bat . h puang tnnh. da cho la khac, phu<111g trlnh da cho ttt<111g duong vm ..ug tri?11 (2). N61 m(>t each phucmg trlnh (3). gOm ba:t phuang trlnh (2) va

Sau day la bai giai vf dl;l 2.

..*Gidi.* Phuong trlnh da cho tuang duang v6i h

*2x +1* >o

1. - '

*{3x2* + 24x + 22 = (2x + 1)2.

148

Scanned by Camscanner

fa CO

*x > -* I

(I) <=> - 2 '

-

*2*

<=>

*x* >--I

2 '

*<:::> x = 2 l.*

*x* -*20x* -21 = O *x* = -1 hoc *x* = 21

Nghim cua phuong trlnh da cho la *x* = 21. o

LB *Giai pht.JtJng trinh Jx 2* + *56x* + 80 = *x* + 20.

Vi dl;I 3.Giai bat phuong trlnh *Jx2* - *3x* - IO < *x* - 2.

*Phan tich.* Diu ki¢n xac djnh cua bat phucmg trinh .da cho la

*x2* - *3x* - I0 0. (1)

00 thay nghi¢m cua bat phuang trlnh da cho phai thoa man dieu ki¢n

*I*

*.X -* 2 > 0. (2)

Vm hai diu ki¢n (1) va (2), bat phucmg trinh da cho tucmg ducmg Oi bAt phucmg trlnh

*x*2- *3x* - 10 < *(x* - 2)2. (3)

Nhu vy' bat phuang trlnh da cho tucmg dttcmg vm h g6m ba bat phucmg

trlnh (I), (2) va (3).

Sau dAy la bai giai vf d9 3.

*Gidi.* &t phucmg trlnh da cho tucmg ducmg v6i h¢

*x2* - *3x* - 10 0

(I) *x* - 2 > 0

*x 2* - *3x* - I 0 < *(x* - 2)2 .

Ta co

*x* < -2 hoc x 5

(I) <=> *x* > 2

*x* < 14

5 *x* < 14.

Tp nghi¢m cua bat phuong trlnh da cho la (5 ; 14). 0

*Gisi b5t phl.lr:Jng trinh Jx z* -*2x* -15 < *x* -3.

149

Scanned by Camscanner



\ • d\t 4.f H;.U h;ll 1'htt\'l\ 11 nh ' · 4 ' · ' · \.

*/' /,,$* I*11 ·It .* n• u "i'n \;\\,' ,tjuh ,,, ,, h•\I I'"'"'''trln h ttl dlll I?\

\ ; .•.\ .' ()

,

( I\

• 'lduht:\ll (· n dn ,,ril\. I;' ''' h:li 1111\'1\l h :vi' :

*l'l'uh1,* I> *'J t I*

.\ J t). \ )

1li n nhk n. n hi m dum n\a l I),, (1) Ii'\ nhi,m ,.\·an h!l't phm1!! rrlnh ,t:1

'hll. Ni'li lll\\I t':\\'h kh h'. ll'\\Uµ 11'\t\lll h\ 'P ntt . h. I phtl\tU lrtnh c.!:t dt<l IU\1!1

hn" n'1 h¢ m hai h. l phu\'lt t 1d1lh l I) *\'i\* {.:n.

rntc))1g *'1* ,,, -

\ *3* o. (:')

\Yi :h.' tlic. U ki¢n ( I) Va (.H. 1';\t plnttl lft llh du d\{) lltrrng c.hn.l'llg, \ lYi h,t

phmW\ tt'lnh

., '- 4.r "<·' - Jr. t4

-

Ifitn nhin (4) k ' rho l I). L'\l tM. nhi\ m drnng "'im h ti h. Iptnrcm!l 1rlnh () \ (4) Ii\ nhi¢m ct\:t h:\t ph&l\lll 1r1nh di1"'h{). Noi "' ch kh• *..:.* tron tnrll11g h9p nay, h\I phtt\ll\ lrtnh c.f,\ d10 lltl'fllg c.hn.m Ve.Yi h g(\m ha i h!\I pllll'Ol1

1rlnh (J) ''(4).

Sau d;ly la bai gi:\ i v d1 *..i.*

*<Jicii.* &'\'t plun.lllg trlnh t:1dw tu\·tng dttrfllg \'CYi

( I) l*f* .r -*..i* .r ..... 0 ho.) (II) J .r-3 0

1

1

Ta co

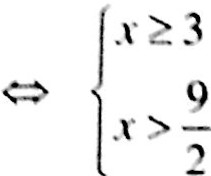
.r -J < O l.r- --tr > (.r -3)-.

(I) *<==>* {.r-0



.\' 3

(II) (::) l. .



f r

2.r 9

.....\_......,,

.\'-< 0.

9



2

Nghi¢m ctia b:lt'

plnr<mg trlnh d,i ho la

*x* 0 vi\ .r 9

-1

Tp nghil)m cun htlt plnmng trlnh da cho la (



:O] u

9 ·+

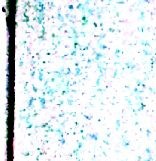
IH4J *Gisi Mt phtJtJng trinh Jx* - 1 *x* + 2.

-2 . c

150

Scanned by CamScanner

. cau hlii 'va ha1ta.n ·



. :;.-· . .. .

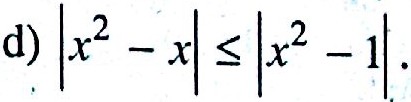
::. ..·65.GiAi cac· phuong trlnh va bat . . , . .

·.. ·.· ··: · Phuong tnnh sau ·

I 41

*:·.* ... . .·. a) *x2* - *5x* + == x·2 + 6 .

*.. .:* , *. X + 5 ·*



b) l x - 11 = 2x -. l ; ·

'

.. . .. ..

: ·. : c)·I-

, . . I· . . .

### ..:. '·

x2. + *x* -1

*2x* + 5 .

. . . . '

·· 66.Giai cac phuong trlnh sau :·

2

. ..

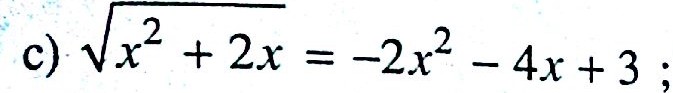
1. 2x

+ *4x* - 1 == *x* + 1 .

'

b) 4x2 + lOlx + 64 = *2( x* + 10) ; ·

d) (x + l)(x + 2) *= x2 + 3x 4.*



··:· . *liul/ng dan.* ) Dt *y* = *x2* + *2x , y* > O,\_ ta duqc phucmg trlnh *y* = - 2 2 +·J.

: ' , *:* d) Vi *(x* + 1)(x + 2) = x2 + 3x + 2 nen dijt *y* = *)x2* + *3x* + 2 , *y* 0, ta dtt<;1c

2

phuong trlnh *y* = y

- 6.

67.Giai cac bat phuang trlnh :

· · \_ a) ;2 + *x ·-* 6 < *x* -1 ;

c) *2x2* - 1 > 1"7". *x* ;

b) .J2 - 1 *2x* - 3 ;

d) *x2* - *5x* - 14 *2x* - 1.

68.Tim tp xac d!nh cua

A" ·h' *6* sau ·

Ol am s .

1. *y* ==

i

*x2*

*+ x + 1* . .

I*2x* -11-*x* 2

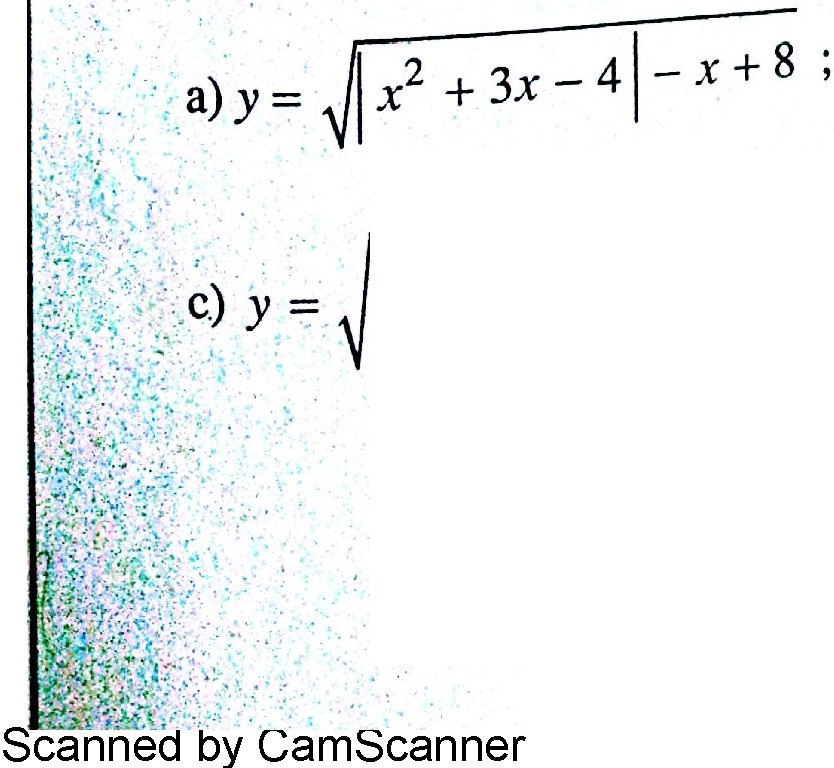
'

·1 1 d) *y* .== *x*

*2*

- *5x* - 14 - *x* '. 3.

-----5: - *xz* + *2x* + 5



*X2* - 7X +

151

~~.,~~



XET DAU PHAN THlJC HOU Tl

BANG PHUONG PHAP KHOANG

Bieu thltc c6 dng *P(x) ,* trong d6 P(.r) va *Q(x)* la nhflng da th(/c, 4t.ICJC g9i la m(>t

* + Q(x)

phan th(tc huu ti.

Ngt.IOi ta chlmg minh dtJc;je rang : Neu cac da thltc *P(x)* va *Q(x)* c6 cac nghi m

XJ • X2, ..., *Xn* d6i m(>t khac nhaU Va Xt < *X2* < ...<.:r,, thitren moi khoang

(-oo ; *x 1). (x 1* ;*x2),•••,(x,,- 1;* x,,), *(x,,* ;+oo),

phan th(tc *P(x)* gilt mot dau kh6ng doi.

Q(x) .

Ap dl:Jng dieu khang <f!nh tren, muon xac dinh dau cua *P( x) trim* moi khoang da

Q(x)

neu,ta chican tinh gia tr! cua phan th(/c ti m¢t diem nao d6 cua khoang.

Pht.1ong phap dt.IQC sl! dl:Jng de xet dau phan thltc huu ti trong cac vi dl) sau day

dt.1QC g9i la pht10ng phap khoang.

Vi dl:J 1.Xet dau rua phan th(tc huu ti

3 2

*Giai.* Tl!*thuc*

*f* (*x* )= 2.r + 3x -*2x* -3

*x* -x-12

2 •

3 2 2 2

2r + 3.r - 2l" - 3 = 2r(x - 1) + 3(.r - 1) = (2r + 3)(x +l)(x - 1)

co' ca,c ngh1'.em Ia' 3 , -1 va. *l* .

. -2-

Mau th(/c la tam thltc bc hai c6 cac nghi m la -3 va 4.

Ta viet phan *thuc* dt.IOidng

*f* ( r) = (*2x* + 3)(*x* +I)(*x* -I)

· *( x +3)( x -4)*

va sap xep cac .nghi m cua tl!th(/c va mau th(/c *cuaj\x )* theo ·thutv tang dan

-3,

3

--,

2

-1, I,4.

Cac nghi m nay chia JR thanh sau khoang

(--00 ; -3), (-3 ;- ). (- ;-t). H; t ), (I ;4) va (4 ; -l<Xl).

152

Scanned by CamScanner

ra xac djnh dau cuaftx) *trlm* m6i khoang da n6u.

I

ra c6 f(O) = 4 > o, do d6 /tx) > O *trtm* khoang (-1 ; 1*) .* Khi *x* qua di m I , chl c6 nh!

th(/C *x* - 1 d6i dau, cac nhj *th(t* khac d u *glCJ* nguyn dAu; do d6/(x) d6i dAu. V1 *vf>y ,*

*fix )* < O trAn khoang < I ;4*).* Khl *x* qua dim -J , chic6 nh!th*Cle x* + I d6id u. do d6*f<x>* d6i di{u. Vl vy *fi x)* < O lriin khoilng (- :-I). Du cUa *ft.x i* trlln cllc khoilng cbn ll,11 dll<;'C xac djnh m¢t each tllong ti! va ta dllgc bang xet dAu sau :

:x) 1 -oo -I + - -Io + o

Vi d&;J 2.Xet dau bi u thuc

4

11 0

+

P(x) = (x2 + 2x + 1)(2x - l )(x2 - *5x* + 4}.

*Giai.* Tam th(tc bc hai *x2* + 2x + I = *(x* + I)2 c6 nghi m kep *x* = -I. Nghi m cua nh! th(/c bc nhat 2x - I la . Nghi m cua tam th(tc bc hai *x2* - *5x* + 4 la I va 4. Ta viet biu thuc da cho dllai dc;1ng

*P(x)* = *(x* + I)\2x - 1)(x - I*)(x* - 4).

Cac nghi m cua *P(x)* sp xep theo th(t ti! ng dan la

-1,

I, I va 4.

2

-

Cac nghim nay chia IR thanh nam khoang

(-oo ; -1), (-1 ; ). ( ; 1). (I ;4) va (4 ; ).

Ta xac dinh dau cua *P(x)* tren moi khoang da neu.

Ta c6 *P(* .

= -4 < 0. 0O d6*fi x)* < o tren khoang (-1 ; \_!\_ ). Khi*x* qua diem .2!. , chic6 nhi

*O)* 2 ·

D d6 *P(x)* dOi dau va ta c6 *P(x)* > 0 tren khoang Tl/Ong thuc 2x - t du1 de1u. o 2

, 4. (.!. ; 1).

*t\f, P(x)* am tren

l. ( 1 . 4) va dtJdng tren khoang (4 ; +oo).

khOGt09 • . .

2 tai diem *x =* -t , nhtJng luon dllong va1 m91 *x :1:-* - t nan khi *x*

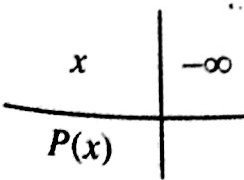
Nhan tl.r *(x* + t ) bng 0 - da

d , *P(x)* < o tren khoang (-oo ; -J ). Ta c6 bang xet

qua diem -1, *P(x)* khong doi u. 00 6

du sau :

0



0

l

2

0

4

+oo

-1

+

0

0

+

153

Scanned by CamScanner



1. Giai cac phttO'llg trlnh va bat phttO'llg trlnh sau :

*x 1* - 2 3x + 4

a) = 2 ; b) *x* \_ 3 ;

.r + l

*2x - 3*

c) 2 1 ;

*x - 3*

1. Giai cac bat phttO'llg trlnh sau :

2

1. I*2x* + 3 I = I4 - *3x* I·

*2*

* 1. \ *.x*

- *5x* + 41 *x2* + 6.r + 5 ;

b) *4x2* + *4x* - I*2x* + I I > 5.

1. Giai cac phttO'llg trlnh sau :

a) *J5x2* - *6x* - 4 = 2 (*x* - I ) ;

1. Giru cac bat phttO'llg trlnh sau :

b) *J x2* + *3x* + 12 = *x 2* + *3x.*

* 1. *vIx*2

+ 6*x* + 8 2*x* + 3 ;

b) *Zx* - 4 > 1·,

*J x2* - *3x* - 10

c) *6 ( x* - *2)( x* - 32) *x2* - *34x* + 48.

*Huflng dan.* c) *Dt y =* (x - *2)( x* - 32) = *J x2* - *34x* + 64 .

1. Giai cac bat phttO'ng trlnh sau :

a) *J x2* - x - 12 2 *x - 1 ;*

b) *Jx2* - 4*x* - 12 > *2x* + 3 ;

c) *.J x + 5* I

<

.

1 -x

1. Tim cac gia ttj ciia *m* sao cho phttO'llg trinh

*x*4 + (1 - 2m*)x*2 +

2 - I = O

*m*

1. Vo nghi¢m ;
2. C6 hai nghi¢m phan bi¢t ;
3. C6 bOn nghim phan bi¢t.
4. Ttm cac gia ttj ciia *a* sao cho phttO'ng trlnh

(a - l)

4 - *ax*2 *+ a*2 - 1 = O

x

c6 ba nghi¢m phn bi¢t.

154

Scanned by CamScanner

*-..\_ L-* drt+,m,=rlfftt'r =

CAu h61va bii tp iin tp chuanu IV .

1. ChUng minh cac bat dang thuc :
   1. I*a* + *b* I < II+ *ab* I v6i I I .

*a.* < 1, I*b* I< I ;

1 . 1 I

b) *n* + 1 + *n.* + 2 + ...+ -> I ,. . *n* E N\* ·

2 - - v01 mo1

*n* 2 . '

*a + b a b*

c) < + - ,. . ,

*b*

l + *a* + *b* 1+ *a* I + vcn m91 *a* 0, *b* 0. Khi nao dang thuc xay ra ?

1. Cht.lng minh cac bat da.ng thuc sau:
   1. *a* + *b* + *c* > + *lb;* + vm *a* o, *b* o, *c* o.Khi nao c6 dfulg thuc ?
   2. *2b2*

*a*

*b2* 2

+ *c*

2 2

+ *c a*

*abc ( a* + *b* + c) vm m9i *a, b, c*

..

E IR. Khi nao c6

da.ng thuc ?

78.Tim gia ttj nho nhat cua cac ham *s6* sau :

1 2

a) /(x) = *x* + - ;

*x*

b) *g( x)* = *x* + 2

*x2* + I

79. lim cac gia ttj cua tham *s6 m* sao cho h bat phUC1ng trlnh sau CO nghim

7 l *3x* 13

*-x - - > - - -*

6 2 2 3

*m2x* + I *m4* - *x.*

80.vm cac gia ttj nao ciia *m,* bat phucmg trlnh (*m2* + 1)*x* + *m* (*x* + 3) + 1 > 0

nghim dung Vffi mQi X E (-l ; Z] ?

81.Giai va bin Juan cac bat phuang trlnh sau :

. .

a) a2*x* + 1 > (*3a* - 2)*x* + 3 ..

'

82. Giai cac bat phtl'<.mg trlnh sau :

a) *x* - 2 > 0 ;

*x2* - *9x* + 20

b) *2x2*

2

+ (*m* - 9)*x* + *m* + *3m* + 4 0..

*2'x2* - l Ox + 14 l

b) 2 . .

*x* - *3x* + 2

155

Scanned by Camscanner

83. Tim cac gia tr!cua *m* sao cho IR la tp

. .

ngh1¢m cua

m6. bflt phLtong trinh sau :

a) *( m* - 4)x2 - *(m* - *6)x* + *m* - 5 0 ;

•

b) ( 2

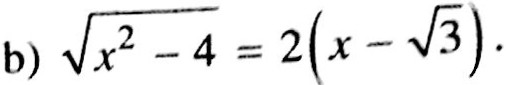
m

- I) 2

+ 2(m + I)x + 3 > 0.

1. Giai cac phucmg trlnh sau :

x

a) I*x2* - *2x* - 31 = 2x + 2 ; 

1. Giai cac bat phucmg trlnh sau :

a) *x2* - *4x* - 12 *x* - 4 ;

b) (*x* - 2) *Jx 2* + 4 x2

- 4 ;

c) *x 2* - 8x 2 (*x* + l) ;

d) *J x ( x* + 3) 6 - x2 - *3x.*

1. Voi cac gia tr!nao cua *a,* cac h¢ bat phuong trlnh sau c6 nghi¢m ?

a){x2 - 5x + 6 < 0 b) {4x + 1 < 7x - 2,

*ax* + 4 < 0 ; *x2* - *2ax* + 1 0.

*Trong cac bai tp tu87 den 89, m6i bai co hon phuong an trd Lili , trong d6 chi c6 m<)t phuong an trd Lili dung, hay ch9n phuong an d6 .*

1. a) Tam thuc bc hai *f ( x)* = *x2* + (l - *.J3)x* - 8 - s.J3 :

\ (A) Duang voi mc;>i *x* E IR ; (B) Am voi mc;>i *x* E IR ;

(C) Am vm mc;>i *x* E (-2 -./3 ; 1+ 2./3) ; (D) Am voi mc;>i *x* E (-oo ; 1).

b) Tam thuc bc hai /(x) = (1 - ..J2\_)x2 + (s -*4J2 )x \_ - 3..fi.* + 6 :

* 1. Duang v mc;>i *x* e JR ;

(C) Duang voi mc;>i *x* E (-4 ; *J2 )* ;

* 1. Duang voi mc;> i *x* E (-3 ;.J2) ;

(D) Am voi mc;>i *x* E IR.

c) Tp xac djnh ciia barn s6 f (x)= (2-.JS)x2 +(1s-1.JS)x+ 25-10.JS lit

(A) 1R ; (B) (-oo ; 1) ; (C) [-5 ; 1] ; (D) [-5 ; *J5* ].

88.a) Tp nghi m cua bat phttcmg trlnh

(3 - 2Ji.)*x2* - 2 {3Ji. -4)*x* + 6(2*J2* - 3) O

ta ·

(A) [-J2 ; 3J2] ;. (B) (-oo ; I ] ; (C) [-1 ;+ex:>) ; (D) [-1 ; 3J2 ].

156

Scanned by CamScanner

b) Tp nghi¢m cua bt phuan , (

g 1

+ J7) x2 + 3x - 14 - 4J7 ?. 0 la

(A) JR ; .

nnh 2

(C) [ -2*Ji* ; 5] ; (B) (-oo ; *-.fi* ] u [2 ; + oo) ;

(D) (-oo ; -*J1*] u [ 1 ;+oo).

c) Tp nghi¢m cua bt phur1n ,

vug tnnh

\_ (*x* - 1)(*x3* - 1)

*x*2 + (1 + *2.Ji)x* + 2 + *.Ji* -< 0

Ia

(A) (-1-J2 ; - ) ;

(C) (-1-*J2* ;- ) u { I } ;

(B) (-1- *)2* ;I] ;

'

(D) [I ; +oo).

1. a) Nghi¢m cua phuang trlnh x2 + I *Ox* - 5 = 2 (*x* - I) la

(A) *x* = 4 ; (B) *x* = 3 - *J6* ;

3

(C) *x* . 3 + *J6* ; (D) *Xi* = 3 + *J6* va X2 = 2.

1. Tp nghi¢m cua bAt phuang trlnh *( x* + 4)(6 -*x) 2( x* + I) Ia

3

(A) [-2 ; 5] ; (B) [ -

; 6] ; (C) [I ; 6] ; (D) (0 ;7].

1. Tp nghi¢m cua bAt phucmg trlnh 2(x - *2)( x* - 5) > *x* - 3 Ia

(A) [-100 ; 2] ;

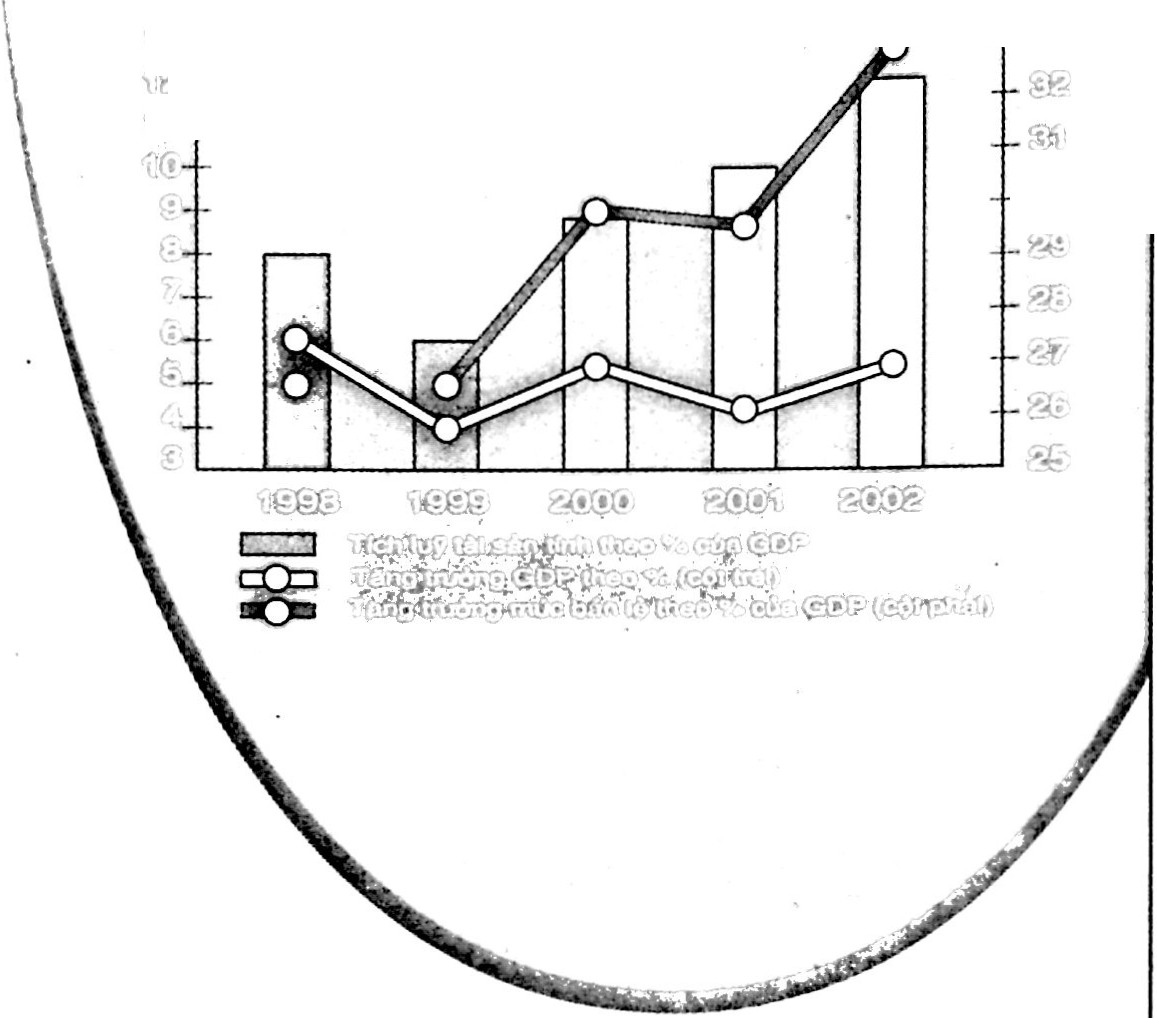
(C) (-oo ; 2] u ['() ;+oo) ;

(B) (-oo ; I ] ;

(D) (-oo ; 2] u (4 + .J5 ; + oo) .

157

Scanned by CamScanner



rJ(<1

**T-HdnG** H-f

I'Ch>(M"if Cf!..rt, .i3!,llif{i· L.·'[[rIl(f · ·

*·ilJ;,1t;r.iit.[!;r:-../i1.0 r.it/rifi;;,*

- { .c

.\

''(

\_. *1'.f*

Trong doisong hi$n nay, **Thong ke** dang ngay cang tr6 nen can thiet va quan tr9ng doivoim9i nganh kinh te xa h(>i.Thong ke giup ta phan tich cac s6 li u m(>t each khach quan va rut ra nhieu thong tin an chua trong cac so li u d6.E)e hieu duqc dieu

d6,chung ta can biet each trinh bay cac so liu thong ke,each tinh cac so d c trung cua cac s6 liu va hieu y ngh a cua chung.

Chae chan chung ta se tim thay cac *(mg* dl)ng cua Thong ke ngay trong cac hot d(>ng cua truong hqc.

158

Scanned by CamScanner



M<)T vAI I<HAI NIM MO DAU

* 1. Thong ke ta gl *'!*

Nhfrng th6ng tin duoi d(;lng s6 li¢u rAt ph6 bien trong khoa h9c va dbi *s6ng.*

Khi d9c m<')t to bao, nghe mQt ban tin tren truyen hlnh, ... chung ta thub'ng bitt gp nhfrng con *s6* th6ng ke.

*Thong ke la khoa hQc ve' cac phU<Jng phap thu thqp, td chuc, trinh bay, phd n*

*tfch va XU li so'li¢u.*

Th6ng ke giup ta phan tfch cac *s6* li¢u m¢t each khach quan va rut ra cac tri thuc, th6ng tin chua dVog trong cac *s6* li¢u d6. Tren co so nay, chung ta moi

c6 th dua ra duqc nhfrng dl;l' bao va quyet djnh dung dn. Vi the, th6ng ke can thiet cho m9i ll;l'C Juqng Jao dC)ng, dc bi¢t rAt cAn cho cac nha quan Ii, hOC;lCh djnh chfnh sach. Ngay ttt dau the ki XX, nha khoa hQc nguai Anh, Oen (H.G.Well) da dl! bao nhu sau : *'Trong mf) t tuung Lai khong xa, kiln thuc*

*thong ke va tu duy thong ke se tr<J thanh mf)t yeu t6' khong thl thieu duqc*

*trong hqc van phd thong cua m6i c6ng dan, gio'ng nhu la khd niing bier d<;> biet viet v{ly".*

2. Miu so liu

u lc1p 7, ta da ..

quen v;:,·1 khai niem dttu hi¢u, don v! dieu tra va gia tri cua

• )"( .\_ 1am

da:u hieu.

v • ·

* .' • *(j'* hoc sinh trong m6i lap hQC 6 cap Trung hQC ph6

Vr du.D d1eu tra ve s · , ,

. , *N e)* nguai dieu tra dn m¢t s6 lap hQC va ghi IC;li si *s6*

th6ng (THPf) cua Ha .1

•

.. • ,,, , . .,

, *:-* Ot doan trfch tll' s6 c6ng tac cua ngum d1tu tra :

cua mOi lap d6. Sau dAy 1a m · ·

'

f

Scanned by CamScanner

1.59

|  |  |  |
| --- | --- | --- |
| STT | Lap | *S6* hQC sinh |
| I | l OA | 47 |
| '-1  3 | lOB l OC | 55  48 |
| 4 | lOD | 50 |
| 5 | 10£ | 50 |
| 6 | l l A | 45 |
| 7 | l l B | 53 |
| 8 | l l C | 48 |
| 9 | l*ID* | 54 |
| 10 | l 1£ | 55 |

Trong vf d t:ren. *diil1 hi¢u x* la *s6* hQC sinh cua m6i lap, *d<Jn vi dilu tra* la m¢t

lap hqc cap THPT cua Ha N¢i, gia tri cua dau hiu *X* (kf hiu *x) a* lap *lOA* la

47. o lap l *OB* la 55. ...

*Mqt t<)p con hilu h{ln cac dan vj dilu tra du{Jc g<;>i la m(Jt mdu. So' phii'n tli cua m<Jt mau dlt<JC g<;>i la kich thuuc mdu. Cac gia tri cua diill hiefu thu dll<JC tren mau dU<JC g<;>i liJ m<Jt mdu sif U (m6i gia trj nhu the'con g<;>i la mqt so'liu cua mdu).*

N€u cac *s6* li¢u trong mAu duqc viet thanh day hay thrum bang thl ta con g<;>i mau

*s6* liu do la *day so'li¢u* hay *bdng so'li¢u.*

Trong vf d1:1 tren, cht1ng ta c6 m¢t mAu Iacac 16p { IOA ; IOB ; IOC ; ...; I ID ; 11£} va mAu *s6* li¢u la ( 47 ; 55 ; 48 ; ... ; 54 ; 55}, kfch thuoc mAu bang 10.

* Neu thlJC hi¢n dieu tra tren m9i don vi dieu tra thl do la *dieu tra toan bq.*

Neu chi dieu tra tren m<'>t mAu thl do Ia *dieu tra mdu.*

IH1l *Ngl.liJi diu tra phai kidm dinh chfit /l.tr:Jng cac hp s{)a cua m(>t nha my che*

*bien sila bang each mo h(>p sOa dd kidm tra.* C6 *thd dieu tra toan b(> hay khfJng ?*

* Diu tra toan b(> d<>i khi kh6ng kha thi vi *s6* luqng cac dcrn vj diu'tra qua Ian hoc vi khi mu6n diu tra thl phai pha huy don vj dieu tra. Chung ta thuemg chi dieu tra mAu va phAn tfch XU If mAu *s6* li¢u thu dugc.

160

Scanned by CamScanner

**cau hoi va bai ta**.**p**

1. *D ,*diu..tr-so co trong moi gia dlnh *b* huy¢n *A,* nguoi ta ch9n ra 80 gia dlnh, th6ng ke so con cua cac gia dlnh d6 va thu dua.c m u *s6* l ie.u sau :

|  |  |
| --- | --- |
| 2 | 4 |
| 2 | 2 |
| 5 | 2 |
| 3 | 5 |
| 4 | 4 |
| 3 | 4 |
| 3 | 2 |
| 1 | 6 |

3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 2 | 0 | 2 | 2 | 3 | 4 | 5 |
| 2 | 1 | 2' | ·2 | 2 | 3 | 2 |
| 3 | 4 | 2 | 2 | 2 | 3 | 2 |
| 1 | 2 | 4 | 4 | 3 | 4 | 3 |

5

7

2

. 3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 4 | 4 | 2 | 5 | 1 | 4 | 4 |
| 1 | 4 | 4 | 2 | 4 | 4 | 4 |
| 3 | 4 | 5 | 6 | 2 | 5 | 1 |
| 5 | 2 | I | 1 | 2 | 4 | 3 |

2

4

1.

* 1. Dau hi¢u va don vj dieu tra *a* cia.y la gl ? Kich thu6'c mfiu la bao nhieu ?

b) Hay viet cac gia tr!khac nhau trong m!u *s6* liu tren.

1. Dieu tra ve di¢n nang tieu th1:1 trong m<)t thang (tfnh theo kW/h) cua 30 gia dlnh *a* m<)t khu pho *A,* ngum ta thu duqc mAu *s6* li¢u sau :

165 85 65 65 70

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 50 | 45 | 100 | 45 | 100 |
| 70 | 141 | 42 | 50 | 150 |
| 57 | 133 | 45 | 65 | 75. |

53

|  |  |  |  |
| --- | --- | --- | --- |
| 100 | 100 | 100 | 90 |
| 40 | 70 | 84 | 59 |

75

Cling hoi tuang tl! nhu trong bai tp 1.

§ .*2.*. . TRINH BAY M¢T MAUs6 LI U

:::::r .

I. Biing ph3n bO tlin sO • t:ln suilt

; . . . ' ve nang suat cua m(>t gi6?ng h1a 1n6'i; dieu tra vien ghi l li

Va d1;1 1.Kht d1eu .ta . ,

0

lua do tren 120thua ru()ng co cling di n tfch Iha.

nang sua:t (t/ha ua *g!o* ieu tra vien nhn thfi'y :

Xem xet mau so h¢U nay'

161

ScaV neff'oy ·camscanner

10 thtra ru(mg cung c6 nang suat 30 ; 20 thtra ru(mg cung c6 nang suat 32 ; 30 thtra ru(mg cung c6 nang suat 34 ; 15 thtra ru(>ng cung c6 nang suat 36 ; I 0 thtra ru(mg cung c6 nang suat 38 ; I 0 thtra ru(>ng cung c6 nang suat 40 ; 5 thtra ru(>ng cung c6 nang suat 42 ; 20 thtra ru(>ng cung c6 nang suat 44.



Trong mau s6 li¢u tren chi c6 tam gia tr!khac nhau Ia : 30 ; 32 ; 34 ; 36 ; 38 ; 40 ; 42 ; 44. M6i gia tr! nay xuat hin m(>t s6 Ian trong mAu so liu. D

*s- Id x dt !ef n. ca moi gici trf trong nuiu st/ li¢u duqc gQi la*

I *tan so cua gw tr! do.*



Ta c6 th tr1nh bay g9n gang mAu s6 liu tren trong *bdng phan bo'tan so'* (g9i

uh la *bdng tan so)* sau day.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Gia trj (x) | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 |  |
| Tan so (n) | IO | 20 | 30 | 15 | 10 | 10 | 5 | 20 | *N =* 120 |

*Bdng 1*

Neu muon biet trong 120 thl.ra ru(>ng, c6 bao nhieu phan tram thi'.ra ru(>ng c6 nang suat 30, 32, ... ta se phai tfnh them tan suat cua m6i gia ttj.

*Tan suatf; cua gia trf X; la tiso'giua tan s(f 11; va kich thuifc mau N*

*·.* = *n;*

*j*

*I* •

*N*

Nguai ta thuang viet tan suat duoi dg phan tram.

136 sung them m(>t hang tan suat vao bang 1, ta nhn duqc *bdng phdn bo' tan*

*so'*- *tan sudt* (gQi tat Ia *bdng tan so'*- *tan su{it)* sau day.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Gia tri | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 |  |
| Tan so | 10 | 20 | 30 | 15 | 10 | 10 | 5 | 20 | N - 120 |
| Tan suat % | 8,3 | 16,7 | 25,0 | 12,5 | 8,3 | 8,3 | 4,2 | 16,7 |  |
|  |
|  | | | | |

CHO Y

*Bdng 2*

162

1. Tren hang tan so, l!guai t thuang danh m(>t o d ghi kich thuoc mau *N.* Kfch thuoc mau *N* bang t6ng cac tan so.
2. C6 th viet bang tan s6 - Hin suat d(;lng "ngang" (nhtr bang 2) thanh bang "d9c" (chuyn hang thanh c(>t nhu bang 3). ....

Scanned by CamScanner

11·Dai S6 10·NC-8

[B}) *Th6ng ke didrn thi mon* Ti

*8*

*b8ng sau dtJy.* 0 *n trong ki thi vlia qua cua* 400 *em hQC sinh cho ta*

|  |  |  |
| --- | --- | --- |
| £>iim bai thl |  | |
| T'n so | T'n suat (%)  1,50  3,75  I0,75 13,25  2 1,25  18,00  ·'··.. r  *f...:'*  ;..(  ...  ... - |
| 0  I  ...  3 | ...  15  43  53 |
| 4 | 85 |
| 5  6 | ...  ·55 |
| 7  8 | 33  18 |
| 9 | 10 |
| 10 | I O |
| N - 400 |

*·.t.* .,... . • *Bang 3*

*£)1c;n t1ep* cac s6 *vao cac ch? tr6ng (...)* d *c(Jt tan s6 va c(Jt tan sua't trong bang* 3.

2. Bang phan bo tan so - tan suat ghep 1(1p

Vi d1;1 2. Ch9n 36 hc;>c sinh nam cua mqt truang THPT va do chieu cao cua ho, ta thu duqc mAu *s6* li¢u sau (dcm vi : cm). .

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 160 | 161 | 16l | 162 | 162 | 162 | 163 | 163 | 163 | 164 164 | 164 | 164 |
| 165 | 165 | 165 | 165 | 165 | 166 | 166 | 166 | 166 | 167 167 | 168 | 168 |

168 168 169 169 170 171 171 172 172 174.

De

trlnh bay mftu *s6* li¢u (theo m¢t tieu chf nao d6) duqc gqn gang, sue tfch,

nht la khi c6 nhieu *s6* li¢u. ta thtfc hin vi¢c *ghep so'li¢u thanh cac /O'p.* d day. ta ghep cac *s6* li¢u tren tha.n h nam theo ac do c6 d<? dai bang nhau. Lap thu nht g6m cac he;> s1 c6 ch1 u cao nam trong don [160 ;162],

I6p thu hai g6m cac hc;>c sinh co ch1eu cao nam trong don [163 ; 165], ... . Khi .

d6, ta se c6 m¢t bang nht.t sau :

|  |  |
| --- | --- |
| L6'p | Tan so |
| [160 ; 162]  [163 ; 165]  [166 ; t 68]  [I69 ; 171]  [172 ; 174] | 6  12  10  5  3 |
|  | *N - 36* |
|  | |

------

*Bdng 4*

163

Scanned by CamScanner

Trong bang 4, *tiln stJ cua m6i l<lp* la *s6* h9c sinh trong 16'p do.

Bang 4 duqc g9i Ia *bdng phtin btf ta'n sfi ghep !Up* (g9i tat la *hdng tdn s6'*

*ghep L(Jp).*

B6 sung m<)t CQt tan suat vao bang 4, ta nhn duqc bang 5 sau

|  |  |  |
| --- | --- | --- |
| Lap | Tan so | Tan suat (%) |
| [160 ; 162] | 6 | 16,7 |
| [163 ; 165] | 12 | 33,3 |
| [166 ; 168] | 10 | -, f)  • •• • t |
| [169 ; 171] | 5 | . I*.*, *.* '  *)* |
| [172 ; 174] | 3 | L |
|  | *N = 36* |  |

*Bdng 5*

Bang 5 duqc g9i Ia *bdng phan botiln so'*- *tti'n suat ghep LU,,* (g9i uh la *bdng*

*tan so'*- *tan sudt ghep l<lp*).

IH2l *Hay dien cac so vao cho trong* (...) c1 *c()t tan suat trong bang* 5.

Trong nhieu truang hgp, ta ghep lap theo cac nua khoang sao cho mut ben phai cua m<)t ni'ra khoang ciing la milt ben trai cua ni'ra khoang tiep theo. ChAng h, trong vf d1:1 2, ta co th ghep cac so li¢u thanh nam 16'p vai cac

nua khoang [159,5 ; 162,5) ; [162,?; 165,5) ; ... . Ta c6 bang sau .

|  |  |  |  |
| --- | --- | --- | --- |
| Lap | Tan so | Tan suat | (%) |
| [159,5 ; 162,5) | 6 | 16,7 |
| [162,5 ; 165,5) | .12 | 33,3 |
| [165,5 ; 168,5) | 10 | 27,8 |
| [168,5 ; 171,5) | 5 | ... |
| [171,5 ; 174,5) | 3 | ... |
|  | *N = 36* |  | |

1. **Biu do**

*Bdng 6*

T1:1c ngu c6 nhilng cau : "Tram nghe kh6ng bng m<)t thay" ; "M6t hinh anh

gia ttj hem ngan lcri n6iII. D trlnh bay mau *s6* li¢u m¢t each lrlf quan sinh

164

Scanned by CamScanner

d(>O·g, *?* nh6' va gAy an tuqng, ngum ta SU dung bieu d6. Sau dAy la m(>t *s6*

bi6u d6 thong d9ng nhat. · ·

a) Biu do tan so, tan suat hinh CQt

Biu do h'inh CQt la m(>t each the hien

. - l

rat *t6t·* bang phAn b6 tAn *s6* (hay t suat) ghep lap.

Vi d1;1 3.Xet bang phAn b6 tan *s6* (bang 4) chi6u cao cua 36 h9c sinh trong vf du 2'.

Hinh 5.1 la bieu do tAn *s6* hinh c6t .the hi¢n bang 4 vm each ve nhu au : Ve hai duang thfulg vu6ng g6c.

Tren duang th!ng nAm ngang (dung lam tru.e s6), ta danh dau cac doa.n xac di.nh 16'p, bat dau tu docµi [160 ; 162] cho tm do [172 ; 174].

Tai m6i docµi, ta dtµlg len m¢t c¢t hlnh

chit nht vm day Ia do<µl d6, con chiu

12

10

8

6

4

2 

160 162 163 165 166 168 169 171 172 174

*Hinh 5.1*

cao bang tan *s6* cua lap ma do<µi d6 12 -



xac d!nh.

Hlnh thu duqc d6 la *bilu do tan s<f* 10

*hinh ct)t.* 0 8

* + D6i vm each ghep lap nhu *b* bng 6: a 6

th hien bang phAn *b6* tAn *s6* bang ieu "1

d h1n.h c6t nhu h'inh 5.2. Trong tnrang 4 .

u · , "kh ho".

hqp nay giii'a cac c(>t kh6ng co e 2 '- .

Chung ta ciing co,the d' ng b,iu d6, hin,h

c6t de the hien bang tAn sua.t ghep

1C1p

. ... ...... .

* + - · ,., *h, h cot* Trong g9i la *bilu sut inhii'* ht se c6 t , h ,y c6t h1nh c .

*do tan*

]59,5 162,5 ]65,5 168,5 171,5 174,5

*Hinh 5.2*

g qp !1a ' - · , (tinh theo %).

ch1eu cao bang Hill suat

[8 *Hay ve bidu a6*

, *§'t hinh c6t thd hin bang* 5.

*t§n su* ·

b) DuOng gAp khllc n s6,

0

*::*c6 khi dlllJc *tht* hin bi\ng m()t bitu d6

Ban hAn b6 tAn so ghep 1*°!'* , ·

g P ,., *h ,c tan so.*

la

khac gQi *duimg ·gap k u*

,

Scanned by CamScanner

165

Vi d1;1 4. Xet bang phan *b6*

-

, \_

trong

tan so

,, d9 2 (

v1

* 4). Duang gap khuc bang

tan

*s6* the hi¢n bg 4 duqc ve nhu sau (h.5.3) : ...

Ta ve

. ... .

ha1 ducmg thang

vu6ng

,,

goc.

Tren

...

duong

thAng

nm ngang (dung

... d.*il* ,

lam

tr9c *SO),* ta danh dau cac diem A1 , *A2, A3, A4, As,*

*b* d6 *A.* la trung km cua

•

d· *A*

*I* - .

doan (hoac nu.a khoa.ng) xa,,c d1.nh la,,p thu,,

*/*. (*1*. \_- 1' 2' ...' 5).T m61 1em i' ta

dtp·lg do·thim·g *AMi* vu6ng g6c·v6i dubng tha•ng na:.m ngang va c6 do· d•ai bang tan

,

soA' cu• a lo,,p tht,r,

*/*• ; Cl;! the• la...

*A* 1M1 = 6, ..., A5*\_* '*M*u *5* - 3· Ve cac do... thang *M 1M 2,*

*M M M M M M* ta duoc mot duc...mg ga-p

A, ,,

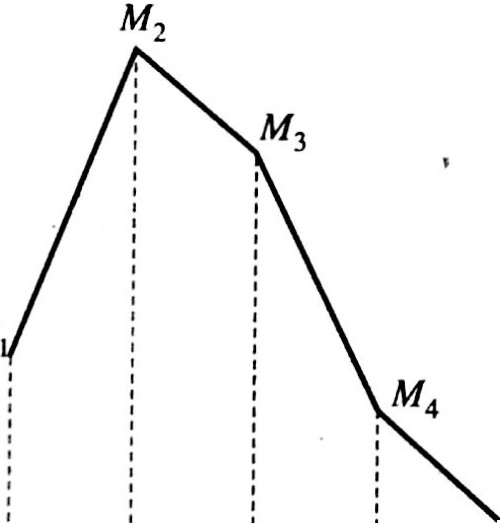
khu,,c.

Do,, ia *duung gap khuc*

2 3, 3 4, 4 5, · . · o

*tiin so·*th hin bang 4.

12



*Ms*

10

8

6 *M*

4

2

I I

I I

:A1 :A2 *A3 A4*

I I I I I I

:As

161 164 167 170 173

*Hinh 5.3*

* + Neu d<) dai dot;in thang *AiMi* dttqc lay bAng Hin suat cua l6p thu *i* thl khi ve cac dot;lfl thang *M 1M 2, M 2M 3,* ..., *M4M 5,* ta dttqc m<)t duang gap khuc gQi Ia

*duhng gap khuc tdn sudt.*

IH4I *Hay din cac so vao ch6 tr6ng trong bang 6 r6i ve dt.JiJng gap khuc tan suat*

*thd hin bang d6.*

1. Bieu do tan suat hlnh qut

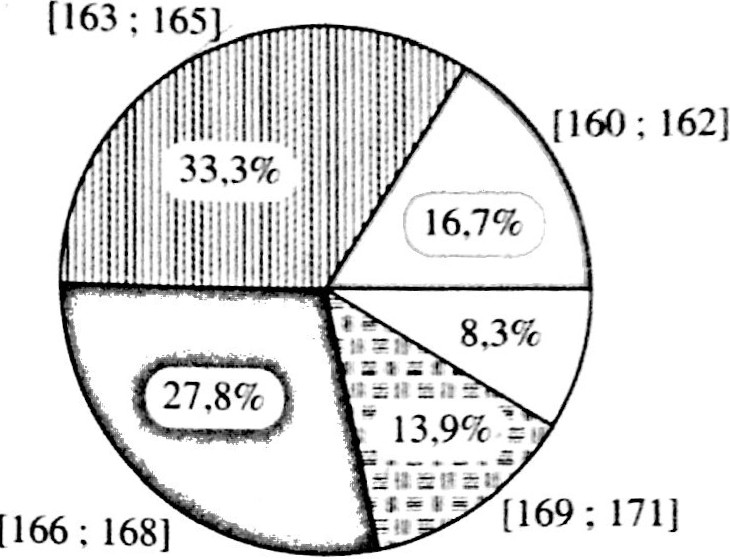
Bieu d6 hlnh qu(;lt rat thfch hqp cho vi¢c th hi¢n bang phAn b6 Hin suat ghep lap. Hinh tron duqc chia thanh nhilng hlnh qut. M6i 16p duqc tuong U'ng vdi m¢t hlnh qut ma di¢n tfch cua n6 ti 1¢ vai tan suat cua lap d6.

166

Scanned by CamScanner

*Vi* d1:1 5. Hinh 5.4 la bieu d - .;·

, , I , O lcln



[172 ; 174J

sull hm 1 qu,\_tt the hi n b'

. ang 5

C:ich ve nlur sau : Ln-n th , *:*

* + '"'P u nhat

It 60 : 1621 chiem \_&.\_

\_ I 16 7

36 - 6 · *o/c*

ctta k ich thu6c 1nfiu . Do do. hlnh

qul;ll tuang t!ng se chiem lhlnh

6

tron. *S6* do g6c cua hlnh qu(;lt nay

a 6 cua , rue la 60 . Ta dung

I, I .. 3600 , , o

*Hinh 5.4*

thtr6c do g6c d d'!Tlg hlnh qu(;lt n6i tren. Tuong ttJ, ta dl!flg hlnh qu<:tt cho cac

lap con l<;ii. Hinh thu dtrqc gQi Ia *bieu dotan suat hinh qu{ll* th hi¢n bang 5. 0

CHO Y

Oic bi U do hlnh cQl va biU do htnh qu t dtrgc Slr dt;mg kh6ng chi nham minh hOC;l bang hlnh anh bang phan bo tan SO - tan suat ghep 16'p ma con dtrqc SU dt;mg r¢ng rai trong vi¢c minh hO<;l cac so li¢u

thong ke *a* cac tlnh huong khac. Xem cac bieu do du& day trfch ra

tu *ThOi bao kinh rlV ief t Nam* 16-12-2002.

-

**l'..111!1**

**NHU CAU VON CHO HTANG GIAO THONG**

.. **E**.**'**;;

*(DOn vt : nghin ti' d6ng)*

-

-

....

....

....

...

...

.....

**:n:**

Im

-

...-

...

•

...

-

--

...

...

-

--

:,

1.......1

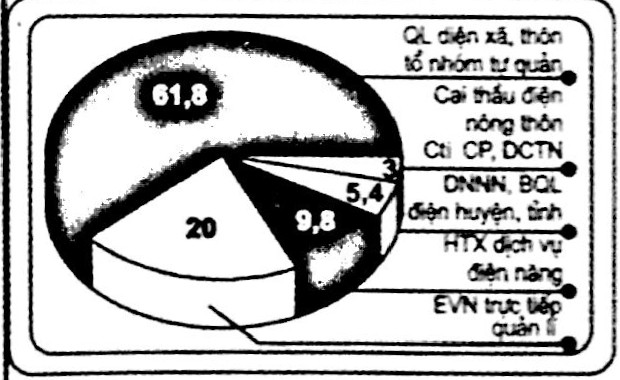
:.........

!.

~~.~~ -".'"'" khi>n9 GT dO th! GT n0ng

.

-----------



*co* cau quan Iikinh doanh

<fi n n0ng than (%)



Scanned by Camscanner

167

**cau hoi wa bai tap**

•

3. Tronv mo·t g1·r·u· 00no

da, hoc sinh. n2um, · ta to..

chu,- mo·.t cuo.c thi du· doan ket

e . '- e . """'

qua Cua 25 tr dang chu · nh:ft. Sau day fa .. phi·u dl\_! d an dung CUa 25

tr ma ban t6 chuc nh duqc :

54 75 121 . 142 15-l 159 l i 1 9 \_o

\_ 11

\_ \_5 \_47 2s1

259 264 278 290 305 3 15 *32\_* 3-- 6 3 -l-0 90.

Hay lp bang tan so - tan suat ghep lap g6n1 sau lap : lap dau tien la do n

(50; 124). lcrp thtr hai fa dO<µl (125 : 199). ... dq dai moi do la 4 ·

1. Mo.t tra.m kim soat giao thong....\_: .g.\_.hi toc do.

tr<µn nhu sau :

km/h cua '")O chiec xe 6 to di qua

53' *4i* 59 *b6* 36 69 83 *11* :r1 5 ST ·60 -8

- -

63 46

031 52 -ss '6f 48 75 60 58 80 -l YJ 60 5 49 63.

Hay lp bang t n so - tan suat ghep lap chinh xac den hang phan nghm ) gom sau lap : lap dau t ien la do(;ln [36 : 43]. lap thti' hai la do (44 : 51]. ... (d<) dai moi do<µl la 7).

1. Dieu tra ve s6 cfia *CD* cua 80 gia dlnh. dieu tra vien thu duo.c bang-

tan suat sau :

tan *6* -

'

|  |  |  |
| --- | --- | --- |
| Lap | Tan so | Tan suat  (% ) |
| [1 : I O]  [11 ; 20]  [21 ; 30] | 5  29  21 | 6.\_ -  - iii... ·.-  - '  -- .... .- |
| (31 ; 40] | 16 | -..--.. |
| (41 ; 50] | 7 | r;. - -  ·"' *:>* |
| [51 ; 60] | 2 | *f'*.,... |
|  | |
|  | N = 80 |  |

* 1. Dien cac *s6* vao cho trong (...) *b* c<)t tan suat.
  2. Ve biu do tan so hlnh c<)t.
  3. ve biu do tan suat hlnh cot.

d) Ve bieu d6 tan suat hinh qut..

168

Scanned by CamScanner

**Luven t'p**

Doanh thu cua 50 Clra han'

6. ? • •

. g cua m(>t cong ti trong m(>t thang nhu sau (dan V! :

m u dong)

95

|  |  |
| --- | --- |
| 88 | 109 |
| 83 | 114 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 147 118 | 148 | 128 | 71 | 93 | 67 | 62 |
| 66 . 156 | 88 | 64 | 49 | IOI | 79 | 120 |

|  |  |  |  |
| --- | --- | --- | --- |
| 120 | 121 | 129 | 1 14 |
| 57 | 103 | 135 | 97 |

166

75 113 155 48 104 112 79 87 88 141 55 123 152 60 83

144 84 95 90 27.

a) Dau hi¢u, don vj dieu tra *a* dAy la gl ?

. b) Lp bang tan so - tAn suat ghep I&p gom bay I&p : l&p dau tien Ia ntra khoag (26,5 ; 48,5), l&p tiep theo la ntra khoang [48,5 ; 70,5), ... (d(> dai moi nua khoang la 22).

c) Ve biu do tAn so hlnh c(>t.

1. M(>t CUQC dieu tra 50 nha nhiep anh nghi¢p du vai cau hoi : "Trong thang

tnrac anh (chi) *sir* d9ng het bao nhieu cu(>n phim ?" cho ket qua nhu sau :

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 5 | 3 | 3 | l | 4 | 3 | 4 | 3 | 6 | 8 | 4 | 5 | 3 | 4 | 2 | 4 | 7 |
| 6 | 5 | 9 | 6 | 6 | 6 | 7 | 0 | 11 | 3 | 12 | 4 | 7 | 14 | 0 | 2 | 4 |
| 4 | 3 | 5 | 15 | 0 | 10 | 4 | 5 | 2 | 3 | 5 | 1 | 8 | 1 | 2 | 12. |  |

* 1. Dau hi¢u, don vj dieu tra *a* day la gl ?

1. Lp bang tan so ghep lap, vai lap dau tien la dO<;iil [0 2], l&p tiep theo la

dO<;tn

(3 ,. 5], ...,IA...,-n cuoi cling Ia do:;in [15 ; 17] (d(> dai moi dO<;iil la 2).

1. Ve bieu do tan *s6* hlnh c(>t.

*y*

• A ,..

,\_

8

* M9t

thu v n thong

kA 0 ngtrb'i den d9c sach vao bu6i toi trong 30 ngay cua

t; s

thang vtra qua nhu sau :

. 4 lap tiep theo Ia do<:lfl [35 , 44], ..., lop cuo1 cung la

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 85 | 81 | 65 | 58 | 47 30 5 1 92 85 42 55 37 31 | 82 63 |
| 33 | 44 | 93 | 77 | 57 44 74 63 67 46 7 52 5 | 47 35. |
| a) LA b? *6*  . ':lP ang tan s | | | | . *;(* hep *lap* (chfnh xac den hang phan tram), vcri 16p  - tan suat g ' . , ,, ' '  ] | |

dau tien la don [25 , 3\_ . ' an la 9). .

do (85 ;94] (d¢ dai mot do.

b) Ve bieu d6

. , h'nh c6t.

tan suat 1 · ·

169

Scanned by CamScanner

bl..... \_C\_A'\_c\_s\_6,\_E>\_A.\_TR\_lfNG\_C\_U

1

\_A\_MAu\_s6\_L\_\_.

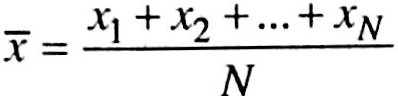
De nhanh ch6ng nam bat duqc nhiing thong tin quan tr<;>ng chtra l;trlg\_ trong

mau *s6* li¢u, ta dua ra m()t vai chi *s6* gqi la *cac sodi)c trung cua mau so liu.*

1. So trung binh
   * Gia sir ta c6 m()t mAu *s6* li¢u kfch thu6'c *N* la *x 1, x2,* ..., *xN.* 0 lap du6i,

ta da biet *so'trung binh* (hay *s6* trung binh c()ng) ciia mftu *s6* li¢u nay' kf hi¢u

la *x* , duqc tfnh bm cong thuc

 (l)

De cho gqn, ta kihiu t6ng Xi + *X2* + ... + *X N* Ia *Lx;* va dqc la "t6ng ciia cac X;

*N*

i=l

vm *i* chy *tit* I den *N" .*V6i kihiu nay, c6ng thuc ( I ) duqc viet g9n la

1 *N*

*x =-Lx; .*

*N* i=t

* + Gia sir mau *s6* li¢u dugc cho du6i dg m()t bang phan *b6* tan *s6* (bang 7) :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Gia tri | Xt *X2* | ... | *Xm* |  |
| Tan s6 | n1 *lli* | ... | *nm* | *N* |

*Bdng 7*

Khi d6, c6ng thtrc tfnh *s6* trung binh (I) *trb* thanh

*x* = n1X1 *+nix2 +...+nmxm* = 1

*N N n;X; ,*

i=l

trong d6 *n;* la Um *s6* cua *s6* li¢u *X;, (i* = I , 2, ..., *m), Ln;* = *N .*

*m*

i=l

(2)

* Gia sir mAu *s6* li¢u klch thuoc *N* duqc cho du6'i dg bang tAn *s6* ghep 16p. Qic

*s6* li¢u duqc chia thrum *m* 16p ll'ng v6'i *m* do (bang 7*a)* hoc *m* 16p ll'ng vOi

170

Scanned by CamScanner

*m* nira khoang (bang 7b). Ta . . . ? ? ., ? ,.

*tr,*

.

vat

,. thu · ... .,. . g9i trung diem *x;* cua doan (hay nua khoang) ung

1<1p *l* 1a *gia df:ii dif n* cua lap d6. .

|  |  |  |
| --- | --- | --- |
|  | | |
| L6'p | Gia tri. da.i die.n | Tan so |
| [a1 *; a2]*  *[ a3* .!*a4]*  .  *[a2m* - I ;*a2m]* | *X t*  ..2  *Xm* | n1  2  .  *nm* |
|  |  | *m*  *N = L n;*  *i* 1 |

*Bdng la*

|  |  |  |  |
| --- | --- | --- | --- |
| L6'p |  | | |
| Gia tri. da.i die.n | Tan s6 | |
| [a1 ; a2)  *[ a2* ..a3)  *[am ; am* +1) | X t  ..2  *Xm* | n1  ..2  *nm*  *m*  *N·= L n;*  i=I |  |
|  |  | · |

*Bdng lb*

Khi d6, so trung blnh cua mAu so li¢u nay duqc tfnh xap xi theo c6ng thuc

1 *m*

*x ::::: -Ln;x; .*

*N* i=t

Vi d1.1 l. Mqt nha thl!C vt hQC do chieu dai cua 74 la cy va thu duqc bang tful

so sau (dan vi : mm) :

|  |  |  |
| --- | --- | --- |
| Lap | Gia tr! di din | Tan so |
| [5,45 ; 5,85)  [5,85 ; 6,25)  [6,25 ; 6,65)  [6,65 ; 7,05)  [7,05 ; 7,45)  [7,45 ; 7,85)  [7,85 ; 8,25) | 5,65  6,05  6,45  6,85  7,25  7,65  8,05 | 5  9  15  19  16  8  2 |
|  |
|  | *N* 74 |
|  | |

171

Scanned by CamScanner

l . 1 ..



Khi d6, chieu dai trung blnh cua 74 la nay xap xi Ia

x :::::: 5 . 5,65 + 9 . 6,05+ ...+ 8 . 7,65 + 2 . 8,05 :::: 6,80 (mm). 0

74

y nghia cua so trung blnh

*So' trung binh cua mau so' !i¢u duqc dung lam dqi di¢n\_cho \_cac s6*

*li¢u cua mau. N6 la m<)t so'd{ic trLlng quan tr9ng cua mau s6 li¢u.*

Chng hn, neu biet rang thOi gian trung blnh de dieu tri khoi b¢h *A* do,i v6i b¢nh nh nam Ia 5,3 ngay, doi vm b¢nh nhan nu ta 6,2 ngay th'i ta c6 the cho rang n6i chung vcri b¢nh *A* thl nh nh nam ch6ng blnh phl;IC hem SO v6i

b¢nh nhan nu.

Tuy nhien , khi cac so li¢u trong mau c6 St! chenh 1¢ch rat Ian d6i vcri nhau thl

*s6* trung binh chua di di¢n tot cho cac so li¢u trong mAu.

Vi d1;1 2. M<)t nh6m 11 hc;>c -sinh tham gia m()t kl thi. *S6* diem thj cua 1 1 h9c sinh d6 duqc sap xep tir thap den cao nhu sau (thang diem I00) :

0 ; 0 ; 63 ; 65 ; 69 ; 70 ; 72 ; 78 ; 81 ; 85 ; 89.

*S6* trung binh Ia

0 + 0 + 63+ .. + 85+ 89 :::::: 61,09.

lI

Quan sat day diem tren, ta thay hAu het cac em (9 em) trong nh6m c6 so diem vuqt s6 trung blnh. Nhu vy, *s6* trung blnh nay khong phcin anh dung trlnh d(J trung blnh cua nh6m. Trong tntb'ng hqp nay, c6 m()t *s6* dc tnrng khac thich

hqp hem d6 Ia *so'trung vf .* . 0

2. So trung vi

*Gid sti ta c6 m<)t mdu st/ li¢u kich thu&c N duqc s&p xep theo tlur*

.

*ft! kho*A*ng gi*•*am . N* ,., *N l* ' *m9*A *t so*,., *l* , ' *s6*,

*eu dUn.g thil*

*N* + l

*eu a*

*e thi*

*li* -

2

*(s(f li¢u dllng chinh giua) gQi la stI trung v*.*i. Neu N ta m6*.*t so'*

*chan, ta Lay trung binh c<)ng cua hal. so' li¢u dUn.g tint N* va

2

-*N*+1 *la'm so*.,.,*trung V!*.*.*

2

*So'trung vi duqc ki hi¢u La M e.*

·112

Scanned by camScanner

. ' "'

*Khi* d6, chieu dai trung blnh cua 74 la nay xap xi ta

*x* 5 . 5,65 + 9 . 6,05 + ...+ 8 . 7,65 + 2 . 8,05 6,so (mm). 0

74

y nghia cua so trung blnh

*So'trung binh cua mau so' lieu dude dung lam dqi di¢n cho cac s6 lie*.*u cua mau. N 6 la mo*.*t so'di*.*ic tng quan trqng cua mau so'li¢u.*

Chlmg h n, neu biet rang thOi gian trung blnh d dieu tr! khoi b¢h *A* do,i v6i bnh nhAn nam J a 5,3 ngay, doi v6'i nh nhan nu la 6,2 ngay thl ta c6 the cho

rng n6i chung vm nh *A* thl nh nhan nam ch6ng blnh ph9c hon so v6i b¢nh nhan nu.

Tuy nhien, khi cac so li u trong mau c6 St! chenh lch rat Ian doi vm nhau thl

*s6* trung blnh chua di di¢n tot cho cac so liu trong mau.

Vi d 2. M()t nh6m 11 hQC -sinh tham gia m()t kl thi. *S6* di m thi cua 11 hQC sinh d6 duqc sap xep tir thap den cao nhu sau (thang diem 100) :

0 ; 0 ; 63 ; 65 ; 69 ; 70 ; 72 ; 78 ; 81 ; 85 ; 89.

*S6* trung blnh Ia

0 + 0 + 63+ .. + 85 +-8,9..., 6I,09.

11

Quan sat day di6m tren, ta thay hu het cac em (9 em) trong nh6m c6 so di m vuc;n so trung blnh. Nhu vy' *s6* trung blnh nay khong phan anh dung trlnh d<) trung binh cua nh6m. Trong truang hqp nay, c6 m(>t so dc trung khac thich

hqp hon d6 la *so'trung vf .* 0

1. So trung v!

*Gid SU ta c6 m<)t mau so'li¢u kich thutJc N duqc sap xep theo th1( tlf khong gidm. Neu N la m<)t so' le thi so' lieu d(tng tlul N* + l

. 2

*(so'li¢u ditng chinh giiia) gqi la so' trung vf. Neu N / a mi)t so'*

*ch&n, ta lay trung binh c¢ng cua hai so' /i¢u difng tint N* va

2

*N* + 1 *lam so'trung vj.*

2 .

*So'trung vj duqc ki hi¢u la M e·*

*· 112*

Scanned by CamScanner

Vi du 3. Dieu t ra ve s6 t , .· .,

* + lQc sinh trong 28 16'p hoc ta duoc mftu s6 li u sau

(sjp xep thco th(r h!tang clan) : . • .

8 39 39 40 40 40 40 40 40 41 41 41

..

42 42

u 43 43 43 44

44 44 44 44 45 45 46

47 47

*S6* lie·u dt!ng thu 14 la 42' dm' 1g thu' 15 Ia' 43 . Do vy, so,tru ng v!. Ia'

*M ,.,* ==

42 + 43

== 42 5 0

< 2 ' .

#### @]

1. *Tinh s6 trung vi cua mau s6 liu trong vi du 2.*

*b) Tinh s6 trung binh cua mau s6 liu trong vi dv 3 va* so *sanh no v{Ji s6 trung vi.*

CHO Y

Khi cac *s6* li¢u trong mau khong c6 Sl;( chenh 1¢ch qua Ian thl so trung blnh va *s6* trung vj xap xi nhau.

IH2I f)o *chieu* cao *cua* 36 *hQC sinh cua m¢t trubng, ta co mau s6 liu sau, s!Jp xep theo thu fl/ tang (don vf : cm) :*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 160 | 161 | 161 | 162 | 162 | 162 | 163 | I63 | I63 | I64 | 164 | 164 |
| 164 | 165 | 165 | 165 | 165 | 165 | 166 | 166 | 166 | 166 | 167 | 167 |
| 168 | 168 | 168 | 168 | 169 | 169 | I70 | I7 I | I71 | I72 | l72 | 174 |

*Tim s6 trung vi cua mau s6 liu nay.*

1. Mot

Cho mot mAu so lieu du6'i dng bang phan b6 tan so. Ta da biet gia ttj c6 tan

soIan hat duqc gi la *mot* cua mau s6 li¢u va kf hi¢u la *M0•*

, d M t a hang ban quan ao thong ke s6 ao so mi nam da ban ra trong

V 4 .?..

I U • 0 Cu ,, d b• *I\ -*

m9t q!uy,th .

eo

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *Ci1* ao (x) | 36 | 37 | 38 | 39 | 40 | 41 | 42 |
| So ao ban dugc (n) | 13 | 45 | 1 10 | 184 | 126 | 40 | 5 |
|  | | | | | | | |

ca,c

*rl* khac nhau va co uoc ang tan so sau

Cv ·

* + ' ? ' tfun nhat fa *ciJ ao nao duqc khach hang mua nhieu nluft.*

D!eu ma cua hang quan ha *ca* ao ma khach hang mua nhieu nhat Iaca 39 (tuc la

g.thong k tre ho tha ) Vay gia ttj 39 fa mot cua mau *s6* li¢u nay. o

g1a tI1 39 c6 tAn so }(111 n · · .

CHU Y

, . , th c6 mot hay

h. - -

n 1eu m6t.

M¢t mAu so lt¢u co .

Scanned by CamScanner

173

Vi d1;1 5.M¢t ci'ra hang ban 6loi qut voi gia tien la I00, 150, 00,35?,400, 500 (dan vj la nghln d6ng). *S6* qut cl.ra hang ban ra trong mua he vua qua duqc th6ng ke trong bang tan *s6* sau



|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | | | | | 400 | 500 |
| Gia tien (x) | 100 | 150 | 300 | 350 |
| Soqut ban duqc (n) | 256 | 353 | 534 | 300 | 534 | 175 |
|  | | |

Ta thay mau *s6* li¢u tren \_c6 hai *m6t* la 300 nghin d6ng va 400 nghin d6ng. D6 la gia tien cua hai loi qut duqc khach hang mua nhiu nhat. D

1. Phuong sai va d() Ich chuan

Vi d1.1 6.Dim trung binh ti.mg mon hQC cua hai hQC sinh n va Binh trong nam hQc vl:ra qua dttqc cho trong bang sau :

|  |  |  |
| --- | --- | --- |
| Mon | Dim cua An | Dim ciia Binh |
| Toan | 8 | 8,5 |
| Vat If | 7,5 | 9,5 |
| Hoa h9c | 7,8 | 9,5 |
| Sinh hQC | 8,3 | 8,5 |
| *N gii* van | 7 | 5 |
| Lich sl'.r | 8 | 5,5 |
| Dja If | 8,2 | 6 |
| Tieng Anh | 9 | 9 |
| Th due | 8 | 9 |
| Cong ngh¢ | 8,3 | 8,5 |
| Giao dl;lc cong dan | 9 | 10 |

IH3I *Tinh didm trung binh (khOng kd h so) .ciia tat ca cac mOn h<;>c cua An va cua Binh. Theo em, bn nao h<;>c kha h<:Jn ?*

Nhln vao bang dim, ta c6 ngay nhn xet la An h9c deu cac mon "' B' h th'

khong.

St!- chenh IA h b.*PI*n

dA ·- , . ?

. , Cun 1n t

cua An thi '

, h.*A.*

c , 1

yng g1ua cac diem

't ?

1 , cua

Bmh

th fl kU.

I

.*m* A

do muc d¢ chenh 1¢ch giiia cac gia trj cua mAu *s6* lieu so v ,..

, . d . , d , *h* . <11 Su trung

1

174

b1nh, ngum ta ua ra hat so c tnmg a *p U(fn.g sai* va *do*.

*L*.*ech c·hua,1.n.*

Scanned by CamScanner

*( ' ' .,* .,

*1*

*ita su*

*"* "" *m(n m6u slf 11/' u kir:'h 1hU'1ir · N I* i I • ,,,. *,r* '.' ;'

*Phu11ng\_ ...ai ula mau sf/ li1 u .nay , kl hii)u /(J* ,/·. *dt111 Jinh hrii*

*f ·ong thu" sau* · '

.\'2 ::: *N*I *N*

*i* I

*<xr* -*x')2. ( 3)*

*Ir*.*on!{ do x /ii* .*•*,*·(J)' ti·unx hL'nh cu*.*a muzu :,1r.' t1·e,u.*

*J*

*Can h(lc hai cua phulfnJ.: sai dw;c WJi Lii di) f#ch chuan.kl hi u W* \ .

... \_ I *N*

•> *-N I <x; -::n2.*

*i* I

*y nghia cua phuung sai va dlJ ch chudn*

Trong cong thuc (3), ta thffy phll<Jng sai la trung binh *cijng* cua binh phu!mg khoang each tl! m6i s6 li¢u *t(1i s6* trung bjnh. Nhu vy.*phur1n;: .\ai a di/ /¢ch chucil1 do muc dl} phan tan cua cac s(/ li¢u trunf( mau quanh si/ / run;.: hlnh. Phllo'ng sai va d¢ /¢ch chuiln canx liln thi d(J phan tan canx Ion.*

CHU Y

C6 th bien d6i cong thuc *<3>* thanh

*(4)*

*Su* d1;1ng cong thtrc (4) thun tin hem trong tfnh toan.

*s2 =-*I*IN xl -*

*N*i=I

I*2* (*LN*

*N i=I*

*X;*J

2

Tro Jai vf du *a* tren, ta hay *tinh* phuang sai va d(> lch chuan diem cac mon

. . *N*

hoc cu,a An va... B'mh. Tr•u'vrlC het' ta tinh cac tc)ng

*L- X ;* va

1. *xl.*

. i=I *i=I*

Tu *s6* Jie.u *a* co.t di m cua An,ta co

II II

* + *A*,*i*

\_- 89'I '· *L- xI* = 725, I I.

* + - . I i=l

*t =*

Tu s6 Iie.u *a* cl>.t diem cua. Blnh, ta co

*L*1 1 *Xi*

*i*.:::I

= 89 ;

*L*II

*i::I*

*X;*2 *=* 750,5.

175

Scanned by Camscanner

Tiep theo, ta the cac ket qua nay vao cong thuc (4) de t'im s2.

Phuong sai va de) 1¢ch chuAn diem cac mon h9c cua An la

2

J

2 725, 11 ( 89,J

0 309 · *S* ::::: *10,3091* - 0,556.

*SA =* - - :::::

11 11 ' ' *A* V

Phuong sai va do. le.ch chuan dim cac mon ho.c cua Blnh fa

2

*S*2*B* = 750,5 -(89)

::::: 2 764 · *SB* ::::: *)* 2,764 ::::: 1,663.

11 II ' '

Ta thay mlic do. "ho.c le.ch" cua Blnh so v6'i An duo.c th hie.n qua vi¢c so sanh hai phtrcmg sai : Phuong sai dim cac mon h9c cua Blnh gap gan 9

< 8,945) lful phtrong sai dim cac mon hQC cua An. Dieu d6 phu hqp v6i

nha.n xet Blnh .ho.c le.ch hon An.

Ta ciing c6 th so sanh d¢ hQc 1¢ch cua Blnh va An thong qua vi¢c so sanh hai do lech chuari. D

Vi¢c tinh cac t6ng *LX;* va *I xl* se nhanh ch6ng neu dung may tfnh bo tui.

*N N*

i=l i=l

(Xem bai d9c them d dtrqc hu&{g dAn chi tiet ve each si'.r dl;}ng may tf nh bo



tui trong tfnh toan th6ng ke).

* Neu so Ii¢u duqc cho du6'i dmg bang phan b6 tan s6 (bang 7) thl phuong sai dugc tfnh *bai* cong thuc

2

2 l *m* 2 1 *m*

*s =-:Ln;X;* - 2 *Ln;X;* .

(5)

*N* i=l *N*

( )

*i=I*

Vi d1;1 7. San ltrgng lua (don v! la t<;t) cua 40 thi'.ra ru<)ng thf nghi¢m c6 cung die.n tfch duo.c trlnh bay trong bang tan s6 sau day.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| San hrqng (x) | 20 | 21 | 22 | 23 | 24 |  |
| Tan so (n) | 5 | 8 | 11 | 10 | 6 | *N = 40* |

1. Tim san ltrgng trung blnh cua 40 thi'.ra ru¢ng.
2. Tfnh phucrng sai va d¢ I¢ch chuan.

*Gidi.* Trtr6c h€t, ta tfnh

*L n;X ;* = 884, *Ln;x?* = 19598.

5 5

i= l i=I

176

Scanned by CamScanner

a) san l tJ<;mg trung blnh cl.ta 40 thl!a ru¢ng la

- 884

#r = -= 22 I (ta).

40 .

b) Theo cong thCrc (5), ta c6 ph ucrng sai fa

2 19598 *( &84 ,1*

*s* = 40 - 40 *)* = I,54.

De) 1¢ch chuAn la s = AA ;:::; J ,24 *( ).*

* Gia su mAu s6 li u duqc cho ducri d(!ng bal ng phan b6 tan "· - Ci1'.' *<i*

li¢u duqc chia thanh *m* lap Ung voi *m* doan (hoac m a khOOngJ. *-":* - gi3 dl;li di¢n cua lap tht! *i* (xem bang 7a,*1b) .·* .

Khi d6, phucrng sai cl.ta mftu *s6* l i u nay co th tfnh :rap xi iheo roog :fLu" rt -t

***Vi* d1;1 8.**Tinh phucrng sai va d¢ 1¢.ch chuan cU.a mau so li dio o *:*

*Gidi.* Ta c6

7

*I nixi* = *50219 ,*

i=l

7

*'n-i* = 3443.385.

*l l*

i=l

2

2 3443,385 - 502,9 ;:::; 0,347.

Vy *5* :::::: 74 742

Do lech chun fa s ::::; Jo,347 ;:::; 0- 589 *(mm* ·

**cau hoi va bai 1'P**

,. *d ,* . *\·e'zi ca"u tinh sotnurg binh. 50 tnm?* -



*a\*, # • ,,

r , *b* ,· , *dum*

*rong cac a1 l{l\_f* , \_-, *den hano phan 1ram* .

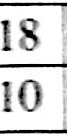
*sai, do Jech chuan (chtnh :tac* ... , \_ \_. , , \_

* + · , h. hoc sinh EDOI Toan thang diem a \_OJ.I\..



* 1. Co I 00 hoc sinh tham dl! ki 1 1 ·

dtrqc cho tong bang sau day.



**Tan** so I I 3 5 8

15 16

19

2 *S =*

1. Tinh *s6* trung blnh. \_ - , ne:hia cua chung.

. ,mot. Neu Y -

1. Tinh *s6* trung V! va do 1ech chufin.

.

1. Tfnh phucmg sa va

1 · ·

l(.OqiS61Q -NC-A

Scanned by CamScanner

-

l

IO. Ng\_um ta chi ,1 9 ct1 khoai tay thanh chin lop n'\n ctr trn kh<\i lm:rng rl'Hl

hung (dO'Il V! la gam). Ta c6 btng ph:ln hC\ 15-n s<) !!h p ll)l' sau d!\y.

|  |  |  |
| --- | --- | --- |
| Lap | | Tan s<> |
| 1 10 : 19} | | 1 |
| 120 : *19]* | | 14 |
| f 30 :39] | | I |
| [40 :49]  f.50 : 59] | | 73  42 |
| [60 :69] | | L |
| [70 : 791 | | 9 |
| [80 : 89] | | 4 |
| l90 :991 | | 2 |
|  |  | *N* = 179 |

Tfnh khi luqng trung blnh Clla m<)t cu khoai tfly. nm phtrong sai va d()

1¢ch chuan.

* + 1. Bang sau day trfch tu s6 theo doi ban hang eta m<)t cfra hang bin xe mny.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| So xe ban trong ngaJ | 0 | 1 | 2 | 3 | 4 | 5 |
| Tan so | 2 | 13 | 15 | 12 | 7 | *3* |

* + - 1. nm *s6* xe trung blnh ban duqc trong m<)t ngay.
      2. Tim phttcmg sai va d<? l¢ch drnAn.

**Love**.**n ta**.**p**

*Trong ctic bai t(lp du&i dd\_\.yeu cdu tinh s6' tnmg hinh. stf tnmg 1·f . pl11m 11g*

*sai* ' *do*.

*te*.*ch chudi1 chinh .rac den hang phdn triim.*

12.

*S6* liu sau day cho ta lai (quy tron) hang thang ciia m<)t cfra hang trong nam 200.?. Dem vi la triu d6ng.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Thang I | | 2 | 3 ! 4 | 5 | 6 | 7 | 8 |  | 9 | 10 | II | *11* |
|  |  |
| Lai | 12 | 15 | 18 I 13 | 13 | 16 | 18 | 14 | 15 | 17 | 20 | 17 |

a) Tim *s6* trung blnh. *s6* trung vj.

b) nm phucmg sai va d<? lch chuan.

13. Mot cfra hang vt Iiu xay dt.rng th6ng ke *s6* bao xi mang ban ra trong 2 ngay

cu6inam 2005. Ket qua nhu sau :

47 54 43 50 61 36 65 54 50 43 62 59 36 45 45 33 53 67

21 45 50 36 58.

178

12·Di so·10·NC·B

Scanned by CamScanner



14.

•) Tim *s6* trung binh, *s6* trung vi.

1. nm phuong sai va d<) 1¢ch ch.uan.

56, h.rcmo khach den tham quan my*A*t d' •

du .

trong

-. ,

duqc hA

. eb?

ke

trong ang

sau day.

1em

h·eh

mot thang

t ung

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| .-- | | | |  | | | | | | | | |
| -Thang | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | I O | II | 12 |
| -s6 khach | 430 | 560 | 450 | 550 | 760 | 430 | 525 | 1 10 | 635 | 450 | 800 | 950 |

* 1. Tun s6 trung blnh, *s6* trung vj.
  2. Tim phuong sai va d(> 1¢ch chuan.

JS. Tren hai con du g *A* va *B,* trm kim soat da ghi J i *t6c* d<) (km/h) cua 30

chiec 6 to tren moi con duang nhtt sau :

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Con duang *A :* 60 | 65 | 70 68 62 75 80 83 | | | | 82 | 69 | 73 | 75 85 72 67 | |
| 88 | 90 | 85 72 63 75 76 85 | | | | 84 | 70 | 61 | 60 65 73 76. | |
| Con duang *B* : 76 | 64 | 58 | 82 72 | 70 | 68 | 75 | 63 | 67 | | 74 70 79 |
| 80 | 73 | 75 | 71 68 | 72 | 73 | 79 | 80 | 63 | | 62 71 70 |

74 69 60 63.

a) nm s6 trung blnh, *s6* trung vj, phttong sai va d<) 1¢ch chuan cua *t6c* d(> 6 t6

tren m6i con duang *A, B.*

b) Theo em th!xe chy tren con duO'ng nao an toan h<111 ?



sl!Dl)NG MAY TfNH s6 TUI TRONG TH6NG KE

. - cu ho *tra rfit* d c lt!c cho vic h9c Th6ng ke. Nho

May tfnh bo tui (MTBT) I cdo:nr ghoc.va de *(mg* dt;Jng.

*er*

MTBT *N* k A *tro* nen e . . . .

*t* \_ t!9

, Thong e a . 0 *SOOMS,* d tfnh so trung binh, phllang sai va

ChAng h(;ln, doi vai may cA ltrin tl.Itheo cac bllCJc sau : lch chu§n, chung ta can 1 , · *6* ke ta *fin*

-·d- tfnh toan th ng ,

(!AODE] [D ,

1) Oiiu Mn,d vao che 9

*D* nhP *s6* li u. ta an

2) Gia Slt mau *s6* li U la x,.*x2* ,..@.. *x,,.*]

QD'f[J \' IDT I

Scanned by Camscanner

.. DT *X2* .• . . *fl*

,,I

179

£)e nhp mau SO l i $U x1 , *X z* ,..., *x,I'* trong d6 *X;* C6 tfin s6 11; , *(i* := I, 2, ···• *rn ).* ta n



x1 I SH I F T I [Jn1 I DT I x2 ISH I ITICJ *n2* [D' ] .··

*x111* IS H FT ICJ *nm* I DT I

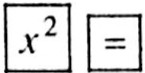
1. Nhp dV li$u xong, de tfnh s6 trung binh *x* I ta an

ISH FT I Is - VARI [] B

1. De tfnh d9 1$ch chuan *S ,* ta an

! SHIFT ! IS - VARI 0 B

1. 8e tfnh phL!Ong sai *s2* (lay binh phL!Ong cua d9 l ch chuan),ta an tiep



Vi d\J 1. Tinh so trung binh, phLtong sai, d¢ 1$ch chuan diem cac mon h<;>c cua An 0

vi dl;l 6,§3.

Sau khi thlfc hi n bl.foe 1, de nhp dv liu. ta an

2) 8 I DT I 7,5 IDT I ... 9 I DT I

1. £)e tfnh trung binh :X, ta an

ISH FT I Is - vARI [] G

Ket qua *x* = 8,I,d6 la so trl:Jng binh can Um. 

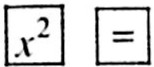
1. De tfnh d9 l ch chuan s, ta an

j SH FT I ls - VARI @] E)

Ket qua *s :::::* 0,555959449, d6 la d9 l ch chuan can tim.

1. De tinh phl./Ong sai *s2 ,* ta an tiep

·1

 I

l

Ket qua ../ 0,309090909, d6 la phL!Ong sai can tlm. 0

Vi d\.I 2. Tfnh so trung.binh,phl/C:Jng sai, d9 l ch chuan cua mau so li u trong vi dl:J 7, §3. I

Sau khi thlfc hi n bU'dC 1, de nhp dv li u. ta an j

2) 20 ISHIFT I[Js IDT 1 21 1 SHIFT I [J s1 DT 1 ... 24 [SHIFT l [] 6 [ DT I

1. D tfnh trung binh *x* ' ta an

ISH FT I ls - VARI [] G

Ket qua *x* = 22,t ;d6 la *s6* trung binh can tim.

180

Scanned by CamScanner

1. f)e tinh de? l ch chuan *s,* ta an

[SHIF'iJ *[§* - VAR] @] G

t(t qua *S* ::::- 1,240967365 , d6 la do. le.ch Chua*11*n *ca*A*n* t'1m.

5) f)e tinh phLIOng sai s2 • ta an tiep

., 0G

Ket qua *s-* 1,54. d6 la phllong sai can tlm. 0

**cau hoi va bait'p on t'p chuang v**

J6. Ch9n phuang an dung trong hon phucmg an tra Joi sau day.

NguOi ta xe'p so can nc}.ng cua I0 h9c sinh theo thu tu tang dAn. *S6* trung vj cua

mfiu *s6* liu nay la : ·

(A) *S6* cAn ng cua h9c sinh thu nam ;

(B) *S6* can m)ng cua h9c sinh thu sau ;

(C) *S6* can nng trung hinh cua em thu nam va thu sau ;

(D) Khong phai cac *s6* tren.

1. Ch9n phuang an dung trong hon phucmg an tra *lai* sau day. De) I¢ch chuan la :
   1. Binh phucmg cua phucmg sai ; (B) M<)t ni'.ra cua phucmg sai ;

(C) can bc hai cua phuang sai ; (D) Khong phai la cac cong thuc tren.

*Trong cac bai ujp tit l8 den 21, yeu cdu tfnh -6- trung binh, so' trung vj,*

*phu<Jng sai, d¢ l¢ch chudn chinh xac den hang pluin triim.*

1. Ngum ta phan 400 qua 1:U'ng thanh a\_m I cn c tren khoi htQ'llg cua chung

(dan vj la gam). Ta c6 bang phan ho tAn so ghep lap sau day.

|  |  |
| --- | --- |
| Lap | Tan so |
| [27,5 ; 32,5)  [32,5 ; 37,5)  [37,5 ; 42,5)  [42,5 ; 47,5)  [47,5 ; 52,5) | 18  76  200  100  6 |
|  | *N - 400* |
|  | |

* 1. Tinh *s6* trung binh. ?

, . ... d"' JAch chuan.

b) T

mh phuang sat va Q

Scanned by Camscanner

18 1

1. MQt nuoi lai xe thuemg xuyen di li gifra hai d!a dim *A* va *B.* Thai gian di (tfnh bang phut) duqc ghi li trong bang phan b6 tan s6 ghep lap sau dAy.



|  |  |  |  |
| --- | --- | --- | --- |
| Lap | | Tan so | |
| [40 ; 44] | | 9 | |
| [45 ;-49] | | 15 | |
| [50 ; 54] | | 30 | |
| [55 ; 59] | | 17 | |
| [60 ; 64] | | 17 | |
| [65 ; 69] | | 12 | |
|  |  | *N =* 100 |  |

* 1. Tfnh thm gian trung blnh ma nguoi d6 di tu *.A* den *B.*
  2. Tfnh phuong sai va d<? lch chuan.

1. MC)t nha nghien CUu ghi li tu6i cua 30 nh nhan mac nh dau mat h()t. Ket qua thu duqc mau s6 liu nhu sau :

21 17 22 18 20 17 15 13 15 20 15 12 18 17 25 17 21 15

12 18 16 23 14 18 19 13 16 19 18 17.

* 1. Lp bang phan bo tan so.
  2. Tfnh so trung blnh va d<? lch chuan.

.C) Tfnh *s6* trung v!va mot.

21.\_ Ngum ta tien hanh phong vn m<)t so ngum ve m()t b¢ phim mm chieu tren truyen hlnh. Ngum dieu tra yeu cAu cho dim b¢ phim (thang dim la 100). Ket qua duqc trlnh bay trong bang phan bo tan so ghep lap sau day.

|  |  |  |
| --- | --- | --- |
| Lap | | Tan so |
| [50 ; 60) | | 2 |
| [60 ; 70) | | 6 |
| [70 ; 80) . | | 10 |
| [80 ; 90) | | 8 |
| [90 ; 100) | | 4 |
|  |  | *N = 30* |

1. Tfnh so trung blnh.
2. Tfnh phucmg sai va d<? lch chuan.

182

Scanned by CamScanner